

7.2 - Synthetic and Long Polynomial Division

Use direct substitution to evaluate $t(a) = -a^5 - a^2 + a + 5$ when $a = -2$

Synthetic Substitution

Synthetic substitution is a method for evaluating a polynomial that uses fewer steps.

- Step 1: Write the value to be evaluated outside and the coefficients in descending order inside.
 Step 2: Bring down the leading coefficient and multiply by the number on the left.
 Step 3: Write the product from the last step under the second coefficient. Add and bring down.
 Step 4: Multiply the sum from the last step by the number on the left.
 Step 5: Repeat for remaining coefficients. The final sum is the value of $f(x)$.

Synthetic Substitution

Example:

evaluate $t(x) = -x^5 - 7x^3 + 3x^2 - 2$ when $x = -1$

$t(a) = -a^5 - a^2 + a + 5$ when $a = -2$

You try! Evaluate $f(x) = 3x^4 - 2x^3 + 4x^2 - 6x - 1$ at $x = -3$.

Remember about Long Division? *(shout out to Mr. Wagonezy!)*

Polynomial Long Division

- a. Divide $f(x) = 3x^4 - 5x^3 + 4x - 6$ by $(x^2 - 3x + 5)$ b. Divide $f(x) = 6m^4 - 12m^3 + m - 2$ by $(m - 2)$

You Try!:

- c. Divide $n^4 + 3n^3 - 7n^2 - 21n$ by $(n + 3)$

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Synthetic Division

Synthetic Division is a method for dividing polynomials that is quicker and more efficient than long division:

Examples:

d. Divide $f(x) = x^3 + 5x^2 - 7x + 2$ by $x - 2$

e. Determine if $(x + 3)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 21$ by using synthetic division. If so, factor completely.

f. Suppose you know that $x = -2$ is a zero of the function $f(x) = x^3 + 2x^2 - 9x - 18$. Find the other zeros.

Application!

Suppose the profit P (in millions of dollars) for a new Algebros T-shirt manufacturer can be modeled by $P = -x^3 + 4x^2 + x$ where x is the number of Bro-Shirts made (in millions). Currently the company produces 4 million shirts and makes a profit of \$4,000,000. Can the company make a lesser number of bro-shirts and still make the same profit?

Now summarize what you have learned!

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Practice 7.2

Evaluate each function at the given value using synthetic substitution.

1. $g(m) = m^3 - 10m^2 + 25m + 2$ at $m = 6$.

2. $f(x) = -x^3 - x + 24$ at $x = 10$.

3. $r(t) = 3t^4 - 8t^2 - 11t + 2$ at $t = -2$

4. $g(x) = 5x^4 + x^2 - x - 41$ at $x = -5$

Divide each polynomial using both long division and synthetic division. Remember, your answers should match ☺

5. $(n^4 + 3n^3 - 9n - 38) \div (n + 3)$

Long Division | *Synthetic Division*

Is $(n + 3)$ a factor of the function?

6. $(x^4 + 16x^3 + 75x^2 + 91x + 49) \div (x + 7)$

Long Division | *Synthetic Division*

Is $n = -7$ a zero of the function?

7.

$$(4a^3 - 36a^2 + 60a + 72) \div (a - 6)$$

*Long Division**Synthetic Division*Is $a = 6$ a zero of the function?

8.

$$(b^4 - 4b^3 + 5b^2 + 8b - 14) \div (b - 2)$$

*Long Division**Synthetic Division*Is $b = 2$ a zero of the function?

Factor each polynomial completely. I'm a nice guy, so I'll give you one of the factors. Your answer for each should consist of 3 binomials.

9. $f(x) = 5x^3 - 18x^2 - 33x - 10$ (One factor is $x - 5$.)

10. $f(x) = 25x^3 - 40x^2 + 17x - 2$ (One factor is $x - 1$.)

Find all the zeros of the given polynomial. I'm still a nice guy, so I'll give you one of the zeros.

11. $f(x) = 15x^3 - 28x^2 + 15x - 2$ (One zero is $x = 1$.)

12. $f(x) = 9x^3 + 3x^2 - 5x + 1$ (One zero is $x = -1$.)

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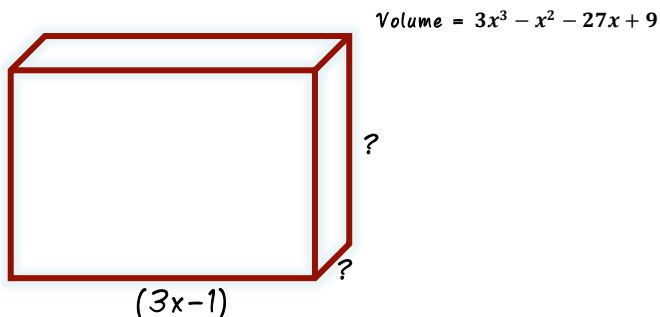
Application 7.2

1. If $f(x) = 6x^3 + 7x^2 - 18x + 5$ and one factor of $f(x)$ is $(x - 1)$, completely factor $f(x)$.
2. Is $m = 7$ a zero of $f(m) = m^4 - 8m^3 + 7m^2$?

Synthetic division clearly simplifies the long division process for dividing by a simple binomial $(x - b)$, but is there a way to use synthetic division when dividing by a linear expression of the form $(ax - b)$ where $a > 1$? Have you noticed that every synthetic division problem so far had a divisor with a leading coefficient of 1.

3. Use long division to divide $6x^3 - 11x^2 - 5x + 12$ by $(2x - 3)$.
4. Use synthetic division to divide $6x^3 - 11x^2 - 5x + 12$ by $(x - \frac{3}{2})$
5. Compare the quotients you calculated in #3 and #4 and the factors $(2x - 3)$ and $(x - \frac{3}{2})$ that you divided by. Now, explain how to use synthetic division to divide by a linear expression of the form $(ax - b)$ where $a > 1$. From this point forward, you should be able to divide synthetically, even if the leading coefficient is not a "1".
6. Find the missing dimensions:

Hint: Divide the volume by $(3x-1)$; then factor!



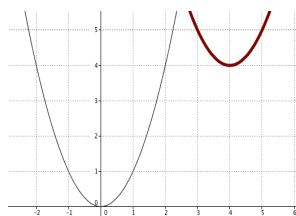
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Algebra Skills

GRAPH

Below, the graph of $f(x) = (x - 4)^2 + 4$ is sketched in bold. Its parent function $f(x) = x^2$ is represented by the thin curve.

- Describe the translation of the parent graph.
- How does the translation relate to the equation?



SIMPLIFY

3. $\sqrt{45} + \sqrt{80} + \sqrt{500}$

4. $(3x^2 - 4x) - (5x - x^3)$

SOLVE

5. Solve: $4(x - 1)(2x - 3) = 0$

6. Factor and solve: $2x^2 + 5x - 3 = 0$

MUTIPLE CHOICE

Determine the number of zeros that are positive integers for the function:

$$f(x) = 6x^3 - x^2 - 12x - 5$$

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) Cannot be determined

SAT Review!

Free Response

What is the remainder when $x^6 - 4x^4 + 4x^2 - 10$ is divided by $(x - 3)$?