

2.1 Remainder Theorem

2.1 PRACTICE

Use the Remainder Theorem to find the remainder for each of the following divisions:

1. $\frac{x^2+3x+1}{x-2}$

$$F(2) = 2^2 + 3(2) + 1$$

$$F(2) = 11$$

2. $\frac{x^3-6x^2-7x+1}{x+4}$

$$F(-4) = (-4)^3 - 6(-4)^2 - 7(-4) + 1$$

$$F(-4) = -64 - 96 + 28 + 1$$

$$F(-4) = -131$$

3. $\frac{16x^4-1}{x+1}$

$$F(-1) = 16(-1)^4 - 1$$

$$F(-1) = 15$$

4. Is $(r - 5)$ a factor of $2r^3 - 15r^2 + 27r - 10$? Find out by using the Factor theorem.

$$F(5) = 2(5)^3 - 15(5)^2 + 27(5) - 10$$

$$F(5) = 250 - 375 + 135 - 10$$

$$F(5) = 0 \text{ yes! BECAUSE } F(5) = 0, (r-5) \text{ IS A FACTOR!}$$

5. Is $(x + 1)$ a factor of $-2x^5 - 4x^4 + x - 10$? Find out by using the Factor theorem.

$$F(-1) = -2(-1)^5 - 4(-1)^4 + (-1) - 10$$

$$= 2 - 4 - 1 - 10$$

$$F(-1) = -13 \quad \text{No!}$$

6. Consider the polynomial function $B(w) = 2w^3 - w^2 - 7w + 6$

a. Verify that $B(-2) = 0$. Since $B(-2) = 0$, what must one of the factors of B be?

$$B(-2) = 2(-2)^3 - (-2)^2 - 7(-2) + 6$$

$$B(-2) = -16 - 4 + 14 + 6$$

$$B(-2) = 0 \text{ YUP!}$$

ONE FACTOR MUST BE $(w+2)$!

b. Find the remaining factors.

$$\begin{array}{r} 2w^2 - 5w + 3 \\ w+2 \overline{) 2w^3 - w^2 - 7w + 6} \\ \underline{2w^3 + 4w^2} \\ -5w^2 - 7w \\ \underline{-5w^2 - 10w} \\ 3w + 6 \\ \underline{3w + 6} \\ 0 \end{array}$$

$$(w+2)(2w^2 - 5w + 3)$$

$$(w+2)(2w-3)(w-1)$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$w = -2 \quad w = \frac{3}{2} \quad w = 1$$

c. State the zeros of B .

$$\underline{\underline{3w+6}}$$

$$\underline{\underline{3w+6}}$$

7. Write a polynomial function in standard form that meets the stated conditions.

a. The zeros are 5 and -4.

$$F(x) = (x-5)(x+4)$$

$$F(x) = x^2 - 5x + 4x - 20$$

$$F(x) = x^2 - x - 20$$

b. The zeros are $-\frac{1}{2}$, 2 and -6.

$$F(x) = (2x+1)(x-2)(x+6)$$

$$= (2x^2 - 3x - 2)(x+6)$$

$$= 2x^3 + 9x^2 - 20x - 12$$

8. Factor completely: $28d^3 + 8d^2 - 7d - 2$

$$\begin{aligned} &4d^2(7d+2) - 1(7d+2) \\ &(7d+2)(4d^2-1) \\ &(7d+2)(2d+1)(2d-1) \end{aligned}$$

9. Factor completely: $20n^4 + 3n^2 - 2$

$$\begin{aligned} &(4n^2-1)(5n^2+2) \\ &(2n+1)(2n-1)(5n^2+2) \end{aligned}$$

10. Is $x = -6$ a zero of $30x^3 + 197x^2 + 100x - 12$?

$$\begin{aligned} F(-6) &= 30(-6)^3 + 197(-6)^2 + 100(-6) - 12 \\ F(-6) &= -6480 + 7092 - 600 - 12 \\ F(-6) &= 0 \\ &\text{yes!} \end{aligned}$$

11. Is $p = -6$ a zero of $3p^3 + 23p^2 + 28p - 12$?

$$\begin{aligned} F(-6) &= 3(-6)^3 + 23(-6)^2 + 28(-6) - 12 \\ F(-6) &= -648 + 828 - 168 - 12 \\ F(-6) &= 0 \quad \text{yes!} \end{aligned}$$

12. Find the value of k so that $\frac{kx^3 + x - k}{x+2}$ has a remainder of 16.

$$\text{Find } F(-2) = 16$$

$$\begin{aligned} k(-2)^3 + (-2) - k &= 16 \\ -8k - k - 2 &= 16 \\ -9k &= 18 \\ k &= -2 \end{aligned}$$

13. Show that $(x + 1)$ is a factor of $19x^{42} + 18x - 1$

$$\begin{aligned} F(-1) &= 19(-1)^{42} + 18(-1) - 1 \\ &= 19 - 18 - 1 \\ F(-1) &= 0 \end{aligned}$$