Name _____ KEY

Semester Exam - Algebra II

Simplify each expression.

Unit 1

1) $-4x(x+4) + x(x+4)$

$-4x^2 - 16x + x^2 + 4x$

$\boxed{-3x^2 - 12x}$

2) $-6(2 - 6x) - 5(x - 4)$

$-12 + 36x - 5x + 20$

$\boxed{31x + 8}$

Solve each equation.

3) $\left(\dfrac{p}{4} - \dfrac{p}{5} = -50p\right)20$

$5p - 4p = -1000p$

$p = -1000p$

$0 = -1001p$ $\boxed{p = 0}$

4) $4|3b + 5| - 2 = -2$

$\qquad +2 \quad +2$

$\dfrac{4|3b+5|}{4} = \dfrac{0}{4}$

$|3b+5| = 0$

$3b + 5 = 0$
$\quad -5 \quad -5$

$\dfrac{3b}{3} = \dfrac{-5}{3}$

$\boxed{b = \dfrac{-5}{3}}$

5) $\left[\dfrac{1}{6}p + \dfrac{19}{6} = -\dfrac{1}{6}\right]6$

$p + 19 = -1$
$\quad -19 \quad -19$

$\boxed{p = -20}$

6) Solve the equation $r = \dfrac{2t}{g}$ for g.

$gr = 2t$

$\dfrac{gr}{r} = \dfrac{2t}{r}$

$\boxed{g = \dfrac{2t}{r}}$

Solve each equation. Write non-integer answers in fractional form.

7) $|-5p| - 1 = 24$

$\quad +1 \quad +1$

$|-5p| = 25$

$-5p = 25$ or $-5p = -25$

$\boxed{p = -5 \text{ or } p = 5}$

8) $1 - 6|5m - 5| = 31$

$-1 \qquad\qquad -1$

$\dfrac{-6|5m-5|}{-6} = \dfrac{30}{-6}$

$|5m - 5| = -5$

$\boxed{\text{No Sol.}}$

Solve each compound inequality and graph its solution.

9) $-2 < 6m + 10 < 34$

$-10 \qquad -10 \quad -10$

$\dfrac{-12}{6} < \dfrac{6m}{6} < \dfrac{24}{6}$

$-2 < m < 4$

10) $-5 + 8r < -37$ or $9r + 8 > 17$

$+5 \qquad +5 \qquad -8 \quad -8$

$\dfrac{8r}{8} \leq \dfrac{-32}{8} \qquad \dfrac{9r}{9} > \dfrac{9}{9}$

$r \leq -4$ OR $r > 1$

Solve each inequality and graph its solution.

11) $-22 \leq 2 - 3m \leq 14$

$-2 \qquad -2 \qquad -2$

$\dfrac{-24}{-3} < \dfrac{-3m}{-3} < \dfrac{12}{-3}$

$8 \geq m \geq -4$

$-4 \leq m \leq 8$

12) $-5 - r > 7$

$+5 \qquad +5$

$-r > 12$

$r < -12$

Solve for the indicated variable in parenthesis.

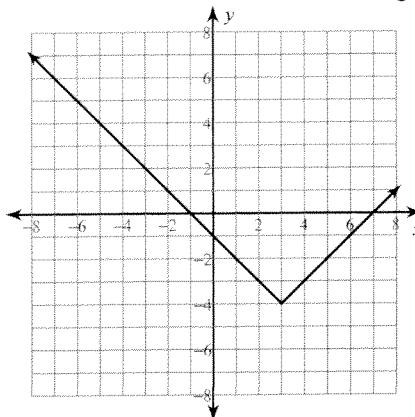13) $k\left(\dfrac{a+b}{k}\right) = 1\cdot k$ (b)

$a+b = k$
$ -a \quad -a$

$\boxed{b = k - a}$

14) $4x - 5y = 9$ (y)

$\begin{aligned} -4x \qquad -4x \end{aligned}$

$\dfrac{-5y}{-5} = \dfrac{-4x + 9}{-5} \quad \rightarrow \quad \boxed{y = \frac{4}{5}x - \frac{9}{5}}$

15) $4x - 5y = 9$ (y)

SAME AS #14

16) Find the domain of the following graph.

$-\infty < x < \infty$

$\boxed{\text{ALL REAL \#'s}}$

Unit 2

17) Find the range.
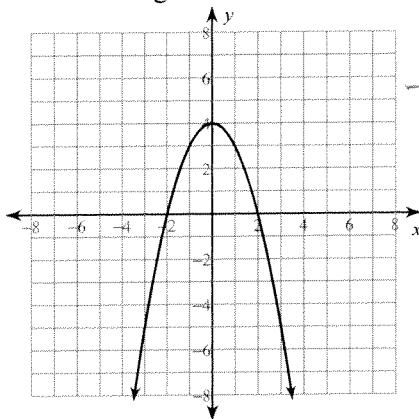
$-\infty < y \le 4$

$\boxed{y \le 4}$

18) Habitat for Humanity builds 5 houses in 6 weeks and then manages to build 15 houses in 14 weeks. What's the average rate of change of houses built per week?

$(6, 5)(14, 15)$

$\dfrac{15-5}{14-6} = \dfrac{10}{8}$

$= 1.25$

$\boxed{1.25 \text{ houses per week}}$

Find the maximum or minimum of the following function.

19) $f(x) = -2(x + 9)^2 + 20$

$(-9, 20) \rightarrow 20$ is a maximum.

Solve.

21) $2(x - 13)^2 - 25 = 175$

$ +25 \quad +25$

$\dfrac{2(x-13)^2}{2} = \dfrac{200}{2}$

$\sqrt{(x-13)^2} = \sqrt{100}$

$x - 13 = \pm 10$
$ +13 \quad +13$

$x = 10 + 13$
$\boxed{= 23}$

or

$x = -10 + 13 \boxed{= 3}$

Evaluate each function.

23) $h(x) = 2x^2 - 6x$; Find $h(-5)$

$h(-5) = 2(-5)^2 - 6(-5)$

$h(-5) = 2(25) + 30$

$h(-5) = 50 + 30$

$\boxed{h(-5) = 80}$

Put the following into standard form.

20) $f(x) = -(x + 4)^2 - 12$

$= -(x+4)(x+4) - 12$

$= -(x^2 + 8x + 16) - 12$

$= -x^2 - 8x - 16 - 12$

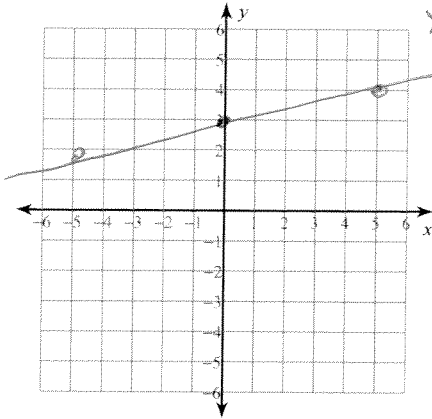$\boxed{f(x) = -x^2 - 8x - 28}$

Evaluate or solve each function.

22) $k(t) = -6t + 14$; Find $k(-5)$

$k(-5) = -6(-5) + 14$

$= 30 + 14$

$\boxed{k(-5) = 44}$

Sketch the graph of each line.

24) $x - 5y = -15$ $\frac{-5y}{-5} = \frac{-x-15}{-5}$
$-x$ $-x$
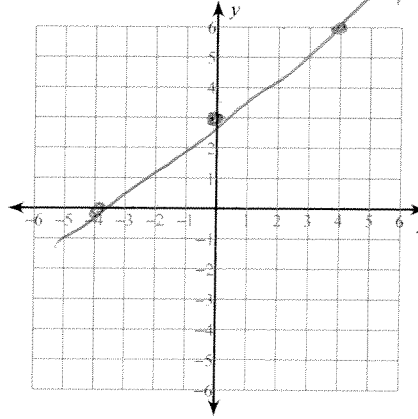$y = \frac{1}{5}x + 3$

Find the x- and y-intercepts, then graph.

25) $3x - 4y = -12$ $\frac{-4y}{-4} = \frac{-3x-12}{-4}$
$-3x$ $-3x$
$y = \frac{3}{4}x + 3$

Write the slope-intercept form of the equation of the line described.

26) through: $(3, -3)$, perp. to $y = \frac{3}{8}x + 3$

$y - y_1 = m(x - x_1)$
$y - 3 = -\frac{8}{3}(x - 3)$
$y + 3 = -\frac{8}{3}x + 8$
$-3 \quad -3$
$\boxed{y = -\frac{8}{3}x + 5}$

27) through: $(3, 3)$, parallel to $y = \frac{4}{3}x - 2$

$y - 3 = \frac{4}{3}(x - 3)$
$y - 3 = \frac{4}{3}x - 4$
$+3 \quad +3$
$\boxed{y = \frac{4}{3}x - 1}$

Write the slope-intercept form of the equation for each.

28) through: $(-4, -2)$ and $(-5, 2)$

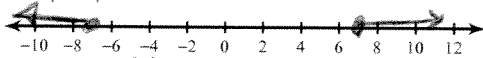$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{-5 - 4} = \frac{4}{-1} = -4$

$y + 2 = -4(x + 4)$
$y + 2 = -4x - 16$
$-2 \qquad -2$
$\boxed{y = -4x - 18}$

Unit 3

Solve each inequality and graph its solution.

29) $|6a| + 10 \geq 52$

$|6a| \geq +42$

$6a \geq +42$ OR $6a \leq -42$
$\boxed{a \geq 7 \quad \text{or} \quad a \leq -7}$

30) $-9|x + 10| - 10 > -46$
$+10 \quad +10$
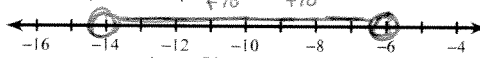
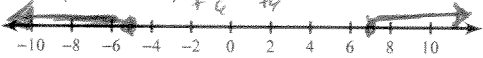$\frac{-9|x+10|}{-9} > \frac{-36}{-9}$

$|x + 10| < 4$

$x + 10 < 4$ AND $x + 10 > -4$
$\boxed{x < -6}$ $\boxed{x > -14}$

31) $2|-7 + 7a| - 6 \geq 78$
$+6 \quad +6$

$\frac{2|-7 + 7a|}{2} \geq \frac{84}{2}$

$|-7 + 7a| \geq 42$

$-7 + 7a \geq 42$ OR $-7 + 7a \leq -42$
$+7 \qquad +7$ $+7 \qquad +7$

$7a \geq 49$ $7a \leq -35$
$\boxed{a \geq 7}$ OR $\boxed{a \leq -5}$

32) $10|10n + 7| - 8 < 62$
$+8 \quad +8$

$\frac{10|10n - 7|}{10} < \frac{70}{10}$

$|10n - 7| < 7$

$10n - 7 < 7$ AND $10n - 7 > -7$
$10n < 14$ $10n > 0$
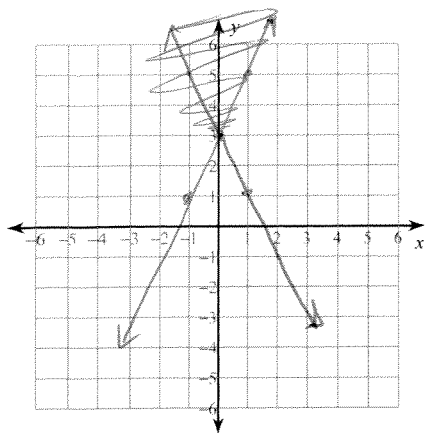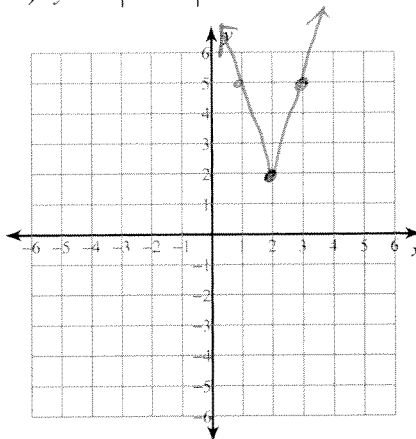$\boxed{n < \frac{7}{5}}$ AND $\boxed{n > 0}$

Graph each equation.

33) $y = -2|x| + 3$

34) $y = 3|x - 2| + 2$

Solve using any method. Unit 4

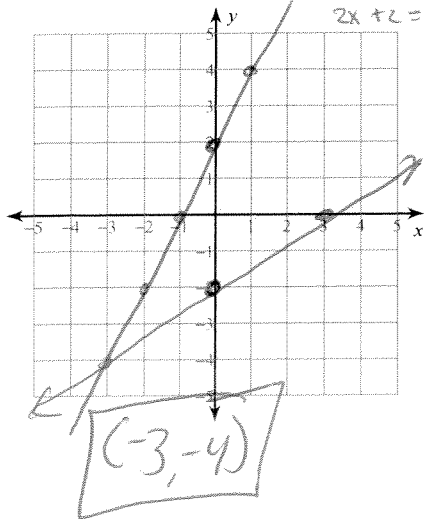35) $1 - y = -x$
$0 = -x + y - 1$
$x = y - 1$

$1 - y = -(y - 1)$
$1 - y = -y + 1$
$1 = 1$ ∟INFINITELY MANY SOLUTIONS

36) $7x + y = -4 \rightarrow y = -7x - 4$
$-2 = -y - x$
$-2 = -(-7x - 4) - x$
$-2 = 7x + 4 - x$
$-2 = 6x + 4$
$-6 = 6x \rightarrow -1 = x$

$-2 = -y + 4$
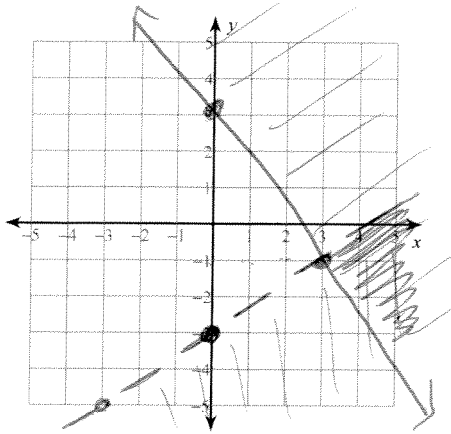$-2 = -y + 1$
$-3 = -y$
$3 = y$ $(-1, 3)$

37) $-3y - 6 = -2x$
$x = -1 + \dfrac{1}{2}y$

$-3y = -2x + 6$
$y = \frac{2}{3}x - 2$
$x + 1 = \frac{1}{2}y$
$2x + 2 = y$

$(-3, -4)$

38) $y \leq -x - 2$
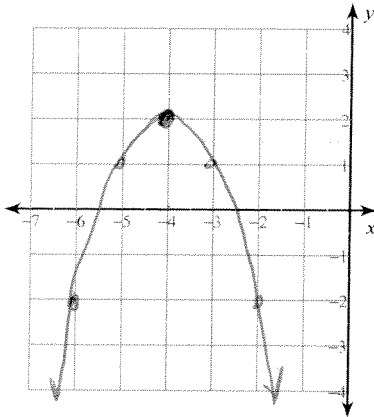$y < 2x + 1$
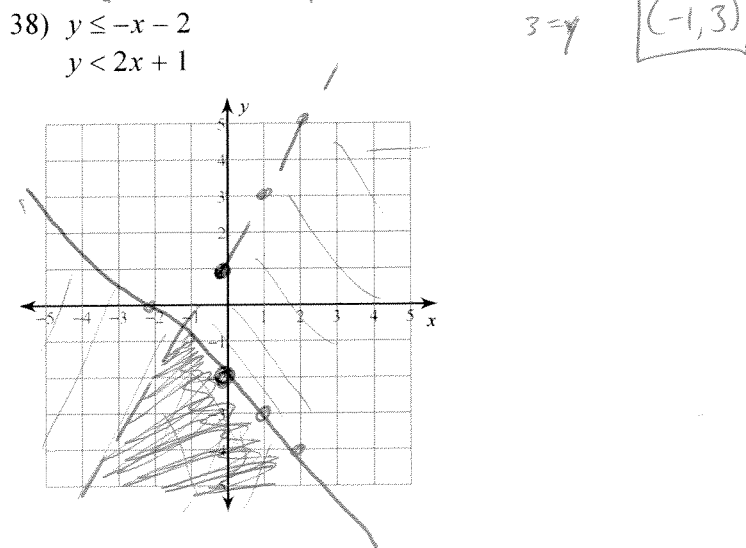
39) $y < \frac{2}{3}x - 3$
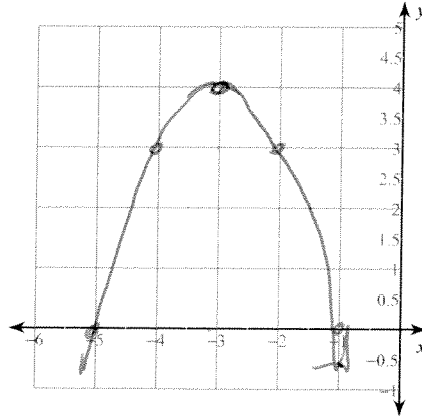
$y \geq -\frac{4}{3}x + 3$

Sketch the graph of each function. Unit 5

40) $y = -(x+4)^2 + 2$

41) $y = -x^2 - 6x - 5$

$x = \frac{-b}{2a} = \frac{6}{2(-1)} = -3$

$y = -(-3)^2 - 6(-3) - 5$
$= -9 + 18 - 5$
$= 4$
$(-3, +4)$

Factor each completely.

42) $3b^4 + 3b^3 - 36b^2$ $3b^2(b^2 + b - 12)$
$3b^2(b-3)(b+4)$

43) $12x^3 + 14x^2 - 48x$ $2x(6x^2 + 7x - 24)$
$x \to 144$
$\to 7$
$2x(6x + \text{16})(6x - 9)$
$2x(3x+8)(2x-3)$

44) $v^2 - 6v + 9$
$(v-3)^2$

45) $12m^2 - 27$
$3(4m^2 - 9)$
$3(2m - 3)(2m + 3)$

Simplify.

46) $\frac{2}{2 + 5\sqrt{2}} \cdot \frac{(2 - 5\sqrt{2})}{(2 - 5\sqrt{2})} \quad \frac{4 - 10\sqrt{2}}{4 - 10\sqrt{2} + 10\sqrt{2} - 25\cdot 2}$ Unit 6

$\frac{4 - 10\sqrt{2}}{4 - 50} = \frac{4 - 10\sqrt{2}}{-46}$

$= \frac{2 - 5\sqrt{2}}{-23}$ or $\frac{-2 + 5\sqrt{2}}{23}$

47) $(2 + 2i) + (-4 + 4i)$
$-2 + 6i$

48) $(7 - 6i) - (-5 + 2i)$
$12 - 8i$

49) $(-7 - 8i)(-5 + 8i)$
$35 - 56i + 40i - 64i^2$
$35 - 16i + 64$
$99 - 16i$

50) $(5 - 5i)^2$
$(5-5i)(5-5i) = 25 - 50i + 25i^2$
$25 - 50i - 25$
$-50i$

51) $\dfrac{2i}{7+9i}\dfrac{(7-9i)}{(7-9i)} = \dfrac{14i-18i^2}{49-63i+63i+81i^2} = \dfrac{14i+18}{49+81}$

$= \dfrac{14i+18}{130} \Rightarrow \dfrac{9+7i}{65}$

52) $\dfrac{5-3i}{-4-i}\dfrac{(-4+i)}{(-4+i)} = \dfrac{-20+5i+12i-3i^2}{16+4i-4i-i^2}$

$= \dfrac{-20+17i+3}{16+1} \Rightarrow \dfrac{-17+17i}{17}$

$= \dfrac{-17}{17} + \dfrac{17}{17}i$

$= -1+i$

Solve each equation by completing the square.

53) $x^2 - 16x - 70 = -4$

$ +70 \quad +70$

$x^2-16x+64 = 66+64$

$\sqrt{(x-8)^2} = \sqrt{130}$

$x-8 = \pm\sqrt{130}$

$x = \pm\sqrt{130}+8$

$= \pm\sqrt{130}+8$

54) $9p^2 + 56 = 18p$

$9p^2 - 18p = -56$

$9(p^2 - 2p + 1) = -56 + 9$

$9(p-1)^2 = -47$

$\sqrt{(p-1)^2} = \pm\sqrt{\frac{-47}{9}}$

$= \pm\frac{\sqrt{47}}{3}i$

$p = \pm\dfrac{\sqrt{47}}{3}$

Solve each equation with the quadratic formula.

55) $4b^2 + 4b = -4$

$4(b^2+b+\frac14) = -4+1$

$4(b+\frac12)^2 = -3$

$(b+\frac12)^2 = \frac{-3}{4}$

$b+\frac12 = \pm\frac{\sqrt{3}}{2}i$

$= -\frac12$

$b = \pm\frac{\sqrt{3}}{2}i - \frac12$

56) $10v^2 - 3v - 9 = -5 + 5v^2$

$5v^2 - 3v - 4 = 0$

$a=5 \quad b=-3 \quad c=4$

$\dfrac{3\pm\sqrt{3^2-4(5)(-4)}}{2(5)}$

$\dfrac{3\pm\sqrt{9+80}}{10}$

$\dfrac{3\pm\sqrt{89}}{10}$

Find the discriminant of each quadratic equation then state the number and type of solutions.

57) $-3x^2 + 5x - 2 = -10$

$a=-3 \quad b=5 \quad c=8$

$\sqrt{25-4(-3)(8)}$

$\sqrt{25+96}$

$\sqrt{121} = 11$

2 Real Solutions

58) $6n^2 - n - 9 = -9$

$6n^2 - n = 0$

$a=6 \quad b=-1 \quad c=0$

$\sqrt{b^2-4ac}$

$\sqrt{(-1)^2-4(6)(0)}$

$\sqrt{1} = 1$

$\pm\sqrt{1}$

2 Real Solutions

Write in vertex form. Find the roots. Graph and label vertex and roots.

59) $y = -x^2 + 4x - 1$

$y+1 = -x^2+4x$

$y+1 = -(x^2-4x+4)$

-4

$y-3 = -(x-2)^2$

$+3 \qquad +3$
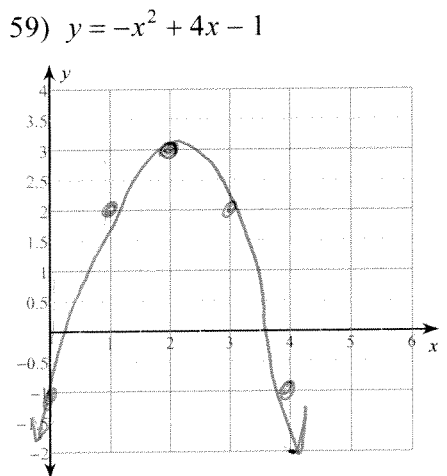
$y = -(x-2)^2+3$

$0 = -(x-2)^2+3$

$-3 = -(x-2)^2$

$+3 = (x-2)^2$

$\pm\sqrt{3} = x-2$

$\pm\sqrt{3}+2 \leftarrow$ Roots

60) $y = \dfrac{1}{2}x^2 + 4x + 6$
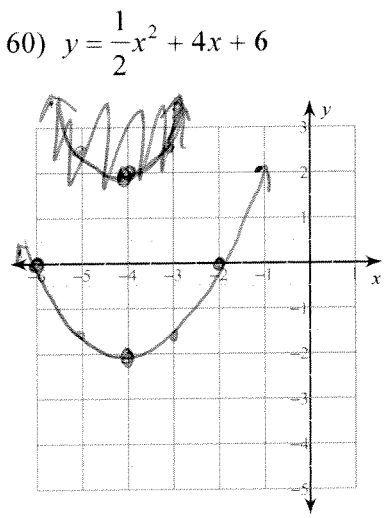
$y-6 = \frac12 x^2 + 4x$

$y-6 = \frac12(x^2+8x+16)$

$+8$

$y+2 = \frac12(x+4)^2$

$y = \frac12(x+4)^2 - 2$

$0 = \frac12(x+4)^2 - 2$

$+2 = \frac12(x+4)^2$

$+4 = (x+4)^2$

$\pm\sqrt{4} = x+4$

$\pm 2 = x+4$

$-4\pm 2 = x$ Roots

$-4+2 = -2$

$-4-2 = -6$

Application and Extension

UNIT 1: Algebros shirts are all the rage! To make an Algebro new shirt, a company charges $60 for a design set-up (one time fee), and then $22 per shirt. You plan on ordering several shirts for your Algebro fan-club.

a. Write an expression that models the total cost for the number of shirts you order including the design-set-up fee.

$$y = 22x + 60$$

b. Use your expression to find the total cost of ordering 20 shirts for your fan-club.

$$y = 22(20) + 60$$
$$y = 440 + 60 = \boxed{\$500}$$

UNIT 2: Timmykat's BARBIE FANPAGE website is BOOMING! A big company offers to come in and pay him to advertise on their page because he is getting so many hits. The CEO of the company says that at 10 hits he'll get $4 and that when he gets to 100 hits he'll get $22.

$$(10, 4) \quad (100, 22)$$

a) What is the average rate of change of dollars per hit that the CEO is offering?

$$\frac{22-4}{100-10} = \frac{18}{90} = \boxed{\$0.20 \text{ per hit}}$$

b) What's the equation of the line for this situation?

$$y = mx + b$$
$$4 = .20(10) + b$$
$$\frac{4 = 2 + b}{-2 \quad -2}$$
$$2 = b$$

$$\boxed{y = 0.20x + 2}$$

c) If Timmykat had 1000 hits in one day how much money would the company give him?

$$y = .20(1000) + 2$$
$$= 200 + 2$$
$$\boxed{= \$202}$$

d) How many hits would it take for Timmykat to make $1000 from the company?

$$1000 = 0.20x + 2$$
$$\frac{-2 \qquad -2}{998 = .20x}$$
$$\boxed{4990 \text{ hits} = x}$$

UNIT 3: Given the piecewise function

$$f(x) \begin{cases} -\dfrac{1}{2}x + 3 & x < 0 \\ 3x - 1 & x \geq 0 \end{cases}$$

Find...

a. $f(4) = 3(4) - 1$
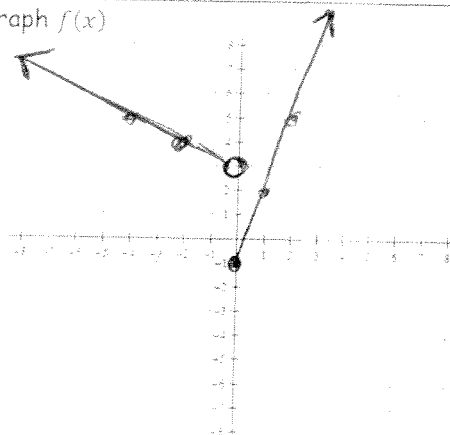$$12 - 1$$
$$\boxed{11}$$

b. $f(-5) = -\frac{1}{2}(-5) + 3$
$$= +\frac{5}{2} + 3$$
$$\boxed{= 11/2}$$

c. $f(0) =$
$$3(0) - 1$$
$$\boxed{-1}$$

Graph $f(x)$

UNIT 4: Sully loves to travel in style... so he invests in an Alge-Blimp! Sullly charges tickets to ride in his Alge-Blimp as either an "Adult" or "Child" ticket. Bean shows up with 7 adults and 7 children and it costs him $196. He had so much fun, he took his whole 3rd period class (14 children and 1 adult) to travel on the Alge-Blimp again and paid $210 (*true story... no hot air here!*). Find the cost of 1 adult and 1 child ticket.

$$-2(7a + 7c = 196)$$
$$\underline{a + 14c = 210}$$
$$\boxed{-14a - 14c = -392}$$

$$-13a = -182$$
$$\boxed{a \, \$14}$$

$$14 + 14c = 210$$
$$14c = 196$$
$$\boxed{c \, \$14}$$

UNIT 5: Bean realizes that he's going to need more room at his house for his ever expanding family. He decides to give his kids more room in their play room. Currently the room is 15 feet by 20 feet and he decides that he'll add 450 square feet to that by adding the same distance, x, to both the length and width of the room.

a) Find the current area of the room.

$$15 \times 20 = 300$$

(square with sides labeled 15 and 20)

b) Write an equation that models adding 450 square feet to the area by adding the same distance x to both the length and the width.

$$300 + 450 = (x+15)(x+20)$$
$$750 = x^2 + 35x + 300$$
$$-750 \qquad\qquad -750$$
$$0 = x^2 + 30x - 450$$

c) Solve the equation and find the new dimensions of the room that Bean is going to build.

$$0 = (x + 45)(x - 10)$$
$$x + 45 = 0 \qquad x - 10 = 0$$
$$x = 45 \qquad x = 10 ✓$$

SO $\quad$ 10 + 15 $\quad$ 10 + 20

$$\boxed{25 \times 30 \text{ ft}}$$
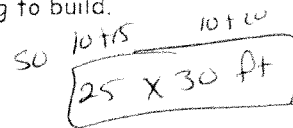
UNIT 6: An object that travels up and down is modeled by the quadratic function below:

$$h(t) = -16t^2 + v_0t + h_0$$

h = height of object (feet), t = time (seconds), v_0= initial velocity of object (ft/sec), h_0= initial height of object (feet)

Bob fires a gun straight up from 7 feet. The bullet is shot straight into air with a velocity of 240 ft/sec.

a. Write the equation that models this.

$$h(t) = -16t^2 + 240t + 7$$

b. What does h(3) mean? Find it.

The height after 3 seconds

c. Find the maximum height of the bullet.

$$t = \frac{-b}{2a} = \frac{-240}{2(-16)} = \frac{-240}{-32}$$
$$t = 7.5$$

$$-16(7.5)^2 + 240(7.5) + 7$$
$$\boxed{907 \text{ feet}}$$

d. When does the rocket hit the ground?

$$0 = -16t^2 + 240t + 7$$
$$a = -16$$
$$b = 240$$
$$c = 7$$

$$\frac{-240 \pm \sqrt{240^2 - 4(-16)(7)}}{2(-16)} = \frac{-240 \pm \sqrt{57600 + 448}}{-32}$$

$$\frac{-240 \pm \sqrt{58048}}{-32} = -.029 \text{ or } 15.03 \text{ sec}$$