Algebra 2

Write your questions and thoughts here!

9.3 – The Number e

Name:_____

RECALL: What is π?

What is i?

Today, we will learn of a new number called ____. It was discovered back in the 17th and 18th century and is called the _____.

WHAT IS e?

Consider the expression $\left(1+\frac{1}{n}\right)^n$. As you increase the value of n, this expression will get closer and closer to the value of e. In other words,

$$\text{As } n \longrightarrow \quad , \left(1+\frac{1}{n}\right)^n \longrightarrow$$

n	1	10	100	100,000	1,000,000
$\left(1+\frac{1}{n}\right)^n$					

$e \approx$

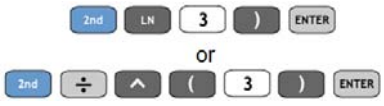
Simplify:

e is a number, but the same rules for variables and exponents still apply.

1. $e^{2x} \cdot e^{5x-1}$
2. $3e^{-x} \cdot 5e^{5-2x}$
3. $\frac{10e^x}{5e^{x-3}}$
4. $e^x(5e^{-3x})^{2x}$

Using a Calculator

5. $e^3 \approx$
6. $e^{-0.2} \approx$



Graphing with the natural base e:

Is $y = a(e)^x$ a growth or decay function?

Is $y = a(e)^{-x}$ a growth or decay function?

9.3 – The Number e

Write your questions and thoughts here!

Financial Fun!

Periodic Compounding Interest: $A =$

Continuous Compounding Interest: $A =$

% increase / decrease: $y =$

____ is the amount of money after _____ years.

____ is the principal (original amount of money).

____ is the interest rate (written as a decimal).

____ is the number of times the interest is compounded (paid) per year.

7. You deposit $1,000 into an account that pays 6% interest compounded continuously. Write a model and then find the balance after 5 years.

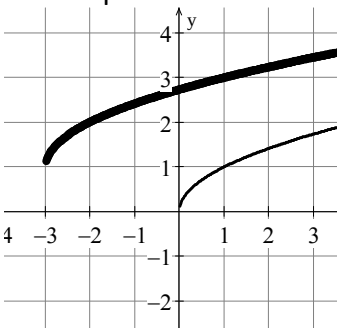
8. You deposit $2,000 into an account that pays 4.8% interest compounded quarterly. Write a model and then find the balance after 10 years.

9. You have a home worth $100,000 and it is increasing in value at 2.5% per year. Write a model and then find the value of the home after 20 years.

Now summarize what you learned!

Algebra Skillz:

1. Below are graphs of $f(x) = \sqrt{x}$ (thin line) and its translation (bold line). Write an equation of the translation.



Simplify the fraction by rationalizing the denominator.

2. $\frac{2}{\sqrt{3}}$

3. $\frac{21}{4\sqrt{7}}$

Solve by factoring.

4. $10x^3 - 110x^2 + 280x = 0$

5. $14x^2 - 35x - 21 = 0$

9.3 Practice – The Number e

Name: _____

For 1-8, simplify. Your answer should contain only positive exponents

1. $e^3 \cdot e^{-5}$
2. $-\frac{e^x}{2e}$
3. $\frac{5e^x}{e^{5x}}$
4. $(2e^{-4x})^3$
5. $-2e^{3x} \cdot e^{-4}$
6. $\frac{e^{2x+7}}{e^{x+3}}$
7. $\frac{e^{x^2-5x+1}}{e^{3x+4}}$
8. $(3e^{x-3})^{2x}$
9. $-\frac{5e^{2x}}{e^{3x}}$
10. $\frac{e^{6x-1}}{e^{x-2}}$
11. $\frac{e^{5x}}{e^3}$
12. $(5e^{2+3x})^2$
13. $4e^{10x} \cdot e^{-8}$
14. $(-3e^{6x})^3$
15. $\frac{e^{x^2+2x-4}}{e^{5x+4}}$
16. $(-2e^{2x+1})^{3x}$

For 17-20, use a calculator to evaluate the expression. Round the result to three decimal places.

17. $4e^2$
18. $-10e^{-2}$
19. $52e^{-4}$
20. $-4e^3$

For 21-24, tell whether the function is an example of exponential growth or exponential decay.

21. $y = 4(e)^{-6x}$ 22. $y = \frac{1}{7}(e)^{3x}$ 23. $y = -\frac{1}{4}(e)^{-x}$ 24. $y = -2(e)^{0.4x}$

For 25 – 32, use one of the three generic models to help you create a specific model for each compounding interest scenario. Then, use your model to calculate the balance for the given amount of time.

Compounding Interest (continuous compounding)	Compounding Interest (periodic compounding)	% increase/decrease per unit of time
$A = Pe^{rt}$	$A = P\left(1 + \frac{r}{n}\right)^{nt}$	$f(x) = ab^x$

25. You deposit $800 in an account that pays 5.7% annual interest compounded continuously. How much will you have after 13 years?

26. Your home is worth $200,000 and increases in value by 2.5% per year. How much will it be worth in 20 years?

27. The value of your baseball cards is currently $320 and increase in value by 0.5% every month. How much will the cards be worth in two years?

28. You deposit $468 into a mutual fund account. It decreases in value by 1% per week for six months. How much do you have after 14 weeks?

29. You deposit $1000 in an account that pays 2% annual interest compounded quarterly. How much will you have after 8 years?

30. You deposit $3500 in an account that pays 8.2% annual interest compounded continuously. How much will you have after 2 years?

31. You deposit $5 in an account that pays 24% annual interest compounded continuously. How much will you have after 50 years?

32. You deposit $111 and it increases by 2% per year. How much money do you have after 15 years?

9.3 Application and Extension

1. Simplify $\frac{(2e^{-3x})^3}{-e^x}$

2. Evaluate and round to three decimals: $14e^{-3}$

3. After winning the lottery, Kelly remembers his Algebra 2 class and the concept of compounding interest. After taxes, he has 2 million dollars he can put into a savings account that compounds continuously.
   a. If the account pays 2% annual interest, how much ***interest*** does he earn after one year?

   b. If the account pays 5% annual interest, how much interest does he earn after one year? Can he quit his job and live off the interest?

   c. If Kelly decides not to spend any of the winnings, and the account pays 5% annual interest, how long will it take before he has $3 million? (*Hint*: Set up the equation, divide both sides by 2 million, graph both sides, find the intersection).

4. Mr. Sullivan's classes have decided to work together and donate money to a local charity benefiting children of math teachers (because we know how painful their life must be). Instead of donating straight to the charity, they decide to put their money in an account and donate the interest every year. After collecting $10,000, they find an account that will pay 12% interest compounded continuously!
   a. Write a ***model*** for the amount of money in the account.

   b. How much will Mr. Sullivan's classes be able to donate **each** year?

5. Air pressure P at sea level is about 14.7 pounds per square inch. As the altitude h (in feet above sea level) increases, the air pressure decreases. This relationship can be modeled by: $P = 14.7e^{-0.00004h}$

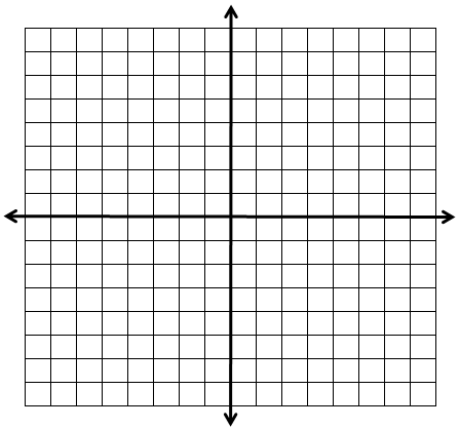
   a. Mount Everest is 29,028 feet at its peak. What is the air pressure at the top of Mt. Everest?

   b. The Dead Sea in Israel is 1,300 feet below sea level. What is the air pressure on the banks of the Dead Sea?

6. Using a calculator, fill in the table for each of the functions below. Then use those coordinate points to sketch a graph of each function.

a) $y = e^x$

x	y
-2	
-1	
0	
1	
2	

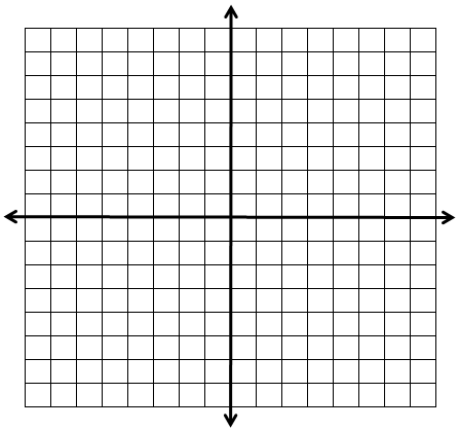


Domain: Range:

Is this function exponential growth or decay?

b) $y = e^{-x}$

x	y
-2	
-1	
0	
1	
2	



Domain: Range:

Is this function exponential growth or decay?

SAT Prep:

1. Simplify: $(2^{x-1})(2^{2-x})^{3x}$

(A) $(2)^{7x-3x^2-1}$
(B) $(2)^{1-3x}$
(C) $(2)^{3x}$
(D) $(2)^{9x^2-3x^3-6x}$

2. If $f(x) = 3(2)^{x-8} + 1$, find $f(6)$.