7.4 – Graphing Polynomial Functions

Write your questions and thoughts here!

To graph, we will need to understand 4 main ideas:

1. What a _____ is
2. What _____ are (Minimum and Maximum)
3. What _____ is
4. What the _____/_____/_____ are

A polynomial function is a function of the form:

$$f(x) = ax^n + bx^{n-1} + cx^{n-2} + \cdots + dx + e$$

where:

a is called the _____ *n* is called _____

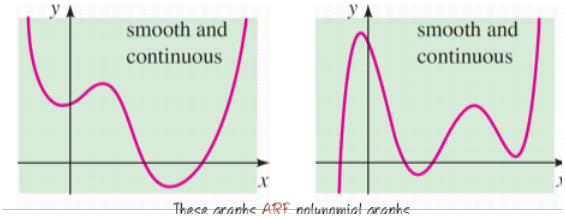
When the function is written with terms in descending order of exponents, we say the function is _____.

We decide whether the function is a polynomial function and state its degree and leading coefficient.

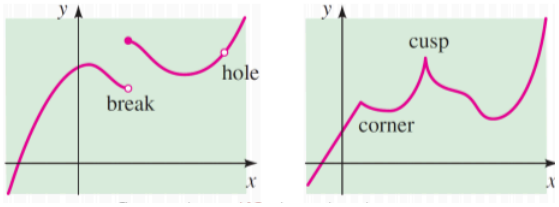
Examples: $r(p) = 2p^3 - 4p^5$ $f(x) = -3x^2 + \frac{1}{x} - x^5$ $f(x) = -3x^2 - x^5$

Graphing Polynomials

The graph of a polynomial function is always a smooth curve; that is, it has no breaks or corners:



These graphs _____ polynomial graphs!



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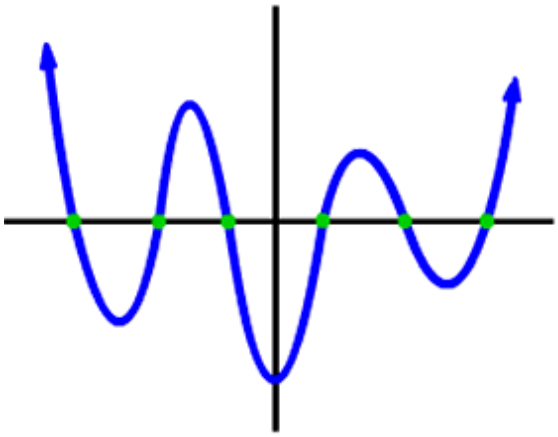


Extrema and Roots (zeros)

_____: Absolute vs. Relative (local)

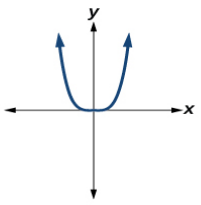
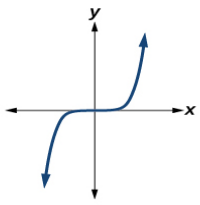
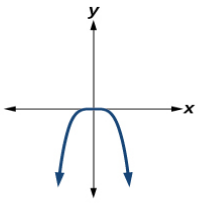
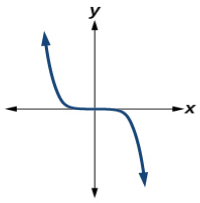
_____: Absolute vs. Relative (local)

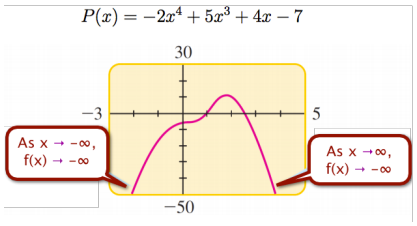
_____: Where crosses x-axis
(also called the x-intercept)

End Behavior

We can see that the **DEGREE** and **LEADING COEFFICIENT** control the shape of the graph!

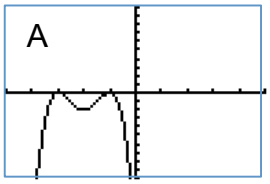
Leading Coefficient

	Even power	Odd power
Positive constant	 $x \to -\infty, f(x) \to \infty$ and $x \to \infty, f(x) \to \infty$	 $x \to -\infty, f(x) \to -\infty$ and $x \to \infty, f(x) \to \infty$
Negative constant	 $x \to -\infty, f(x) \to -\infty$ and $x \to \infty, f(x) \to -\infty$	 $x \to -\infty, f(x) \to \infty$ and $x \to \infty, f(x) \to -\infty$

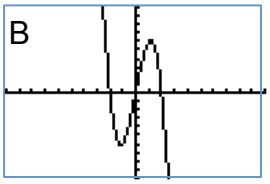


Match the function to the graph using its ***end behavior***.

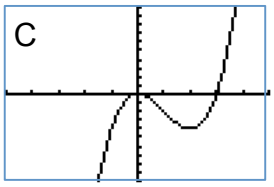
1. $f(x) = x^3 - 3x^2$



2. $f(x) = -2x^3 + 8x$



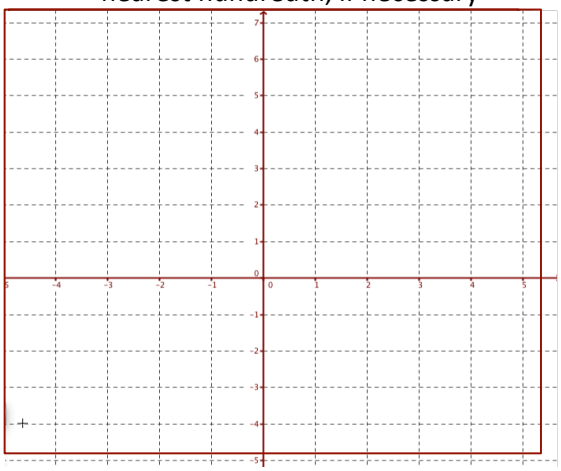
3. $f(x) = -2(x+3)^2(x+1)^2$



Examples:

Let's look at $f(x) = (x+1)(x-2)^2$ or in Standard Form: _____

Graph the function. Label all extrema, zeros, intercepts and end behavior. Round to the nearest hundredth, if necessary



x	f(x)

You try!

Graph the function. Label all extrema, zeros, intercepts and end behavior. Round to the nearest hundredth, if necessary.

$$f(x) = 2x^4 - 5x^2 + 2x$$

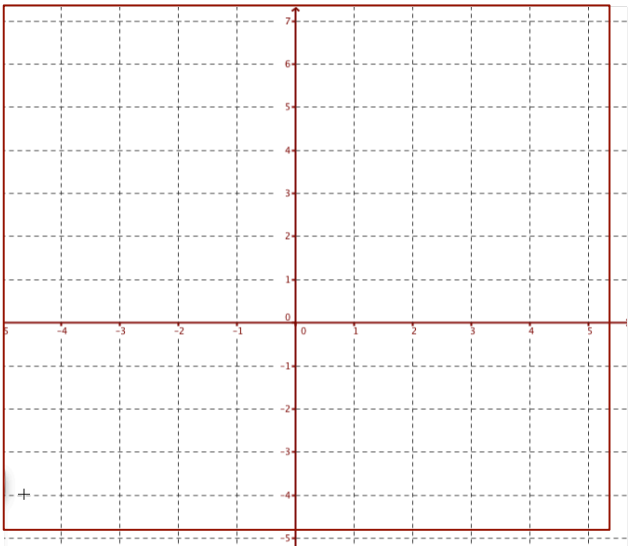
x	f(x)

Zeros:

y-intercept:

Extrema:

End Behavior:



Now summarize what you have learned!

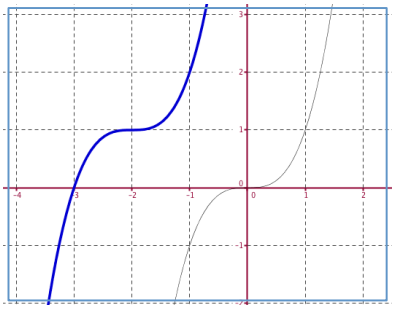
Algebra Skillz

GRAPH

Below, the graph of $f(x) = (x-4)^3 + 4$ is sketched in bold. Its parent function $f(x) = x^3$ is represented by the thin curve.

1. Describe the translation of the parent graph.

2. How does the translation relate to the equation?



SIMPLIFY

3. $-4\sqrt{20} + 2\sqrt{80} + \sqrt{45}$

4. $-2\sqrt{5}(1 - 2\sqrt{5})$

SOLVE

5. Solve:

$$3x^2(x+7)(7x-15) = 0$$

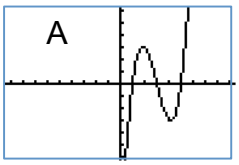
6. Factor and solve.

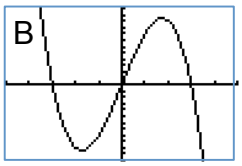
$$x^3 - 2x^2 + x = 0$$

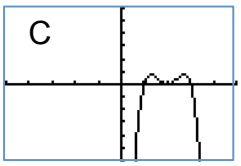
Practice 7.4

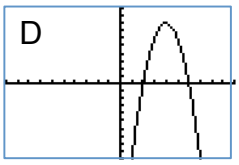
For each of the following, use the end behavior and x-intercepts to match the equation to its graph.

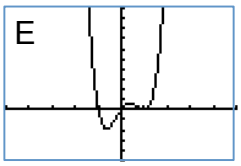
1. $f(x) = x$
2. $f(x) = (x-1)(x-3)(x-5)$
3. $f(x) = -x^3 + 9x$
4. $f(x) = -3(x-1)(x-2)^2(x-3)$
5. $f(x) = -2x^2 + 16x - 24$
6. $f(x) = 3x^4 - 3x^3 - 3x^2 + 3x$

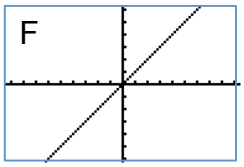




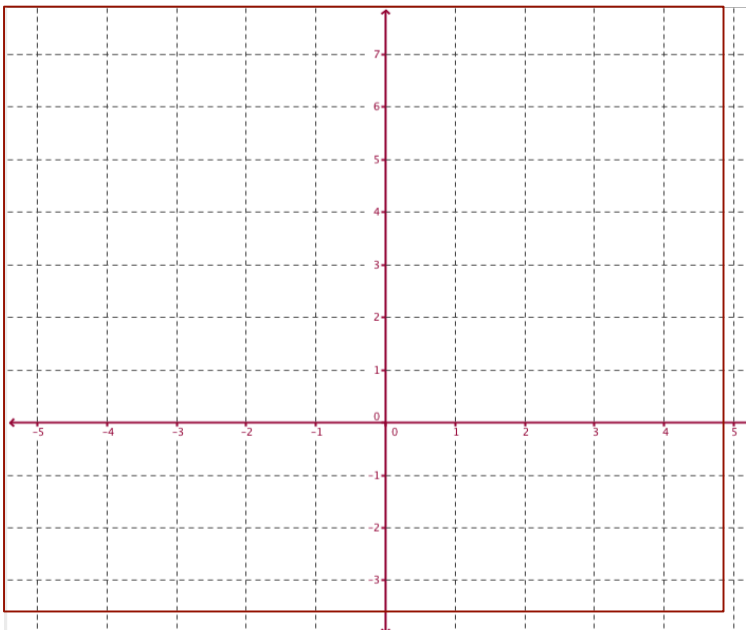








7. Graph the function. Label all extrema, zeros, intercepts and end behavior. Round to the nearest hundredth, if necessary.



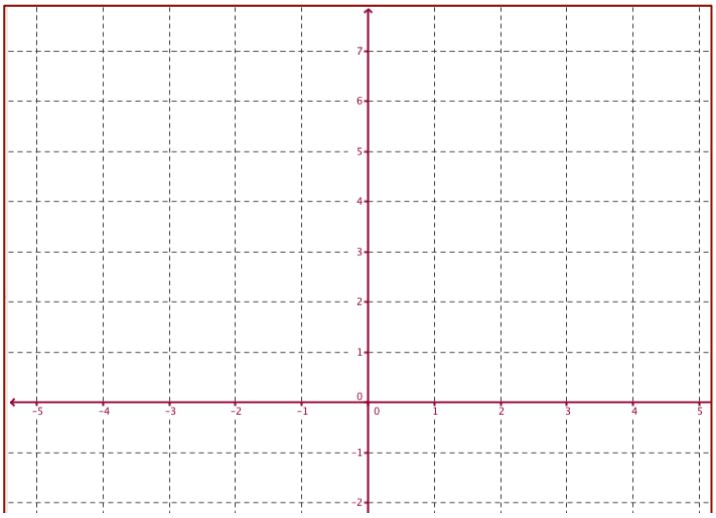
$$f(x) = -x^4 + 5x^2 - x - \frac{1}{2}$$

x	f(x)

8. Graph the function. Label all extrema, zeros, intercepts and end behavior. Round to the nearest hundredth, if necessary.

$$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 3x + 2$$

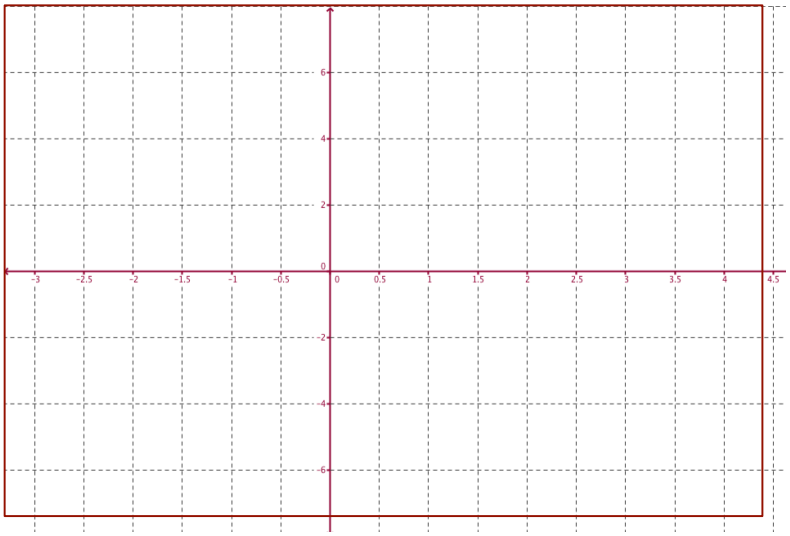
x	f(x)



9. Graph the function. Label all extrema, zeros, intercepts and end behavior. Round to the nearest hundredth, if necessary.

x	f(x)

$f(x) = x^5 - 6x^3 + 5x$



Application 7.4

1. Graph the function $f(x) = x^3 + 3x^2 - 6x - 6$ in your calculator. Describe extrema, zeros, intercepts and end behavior. Round to the nearest hundredth, if necessary.

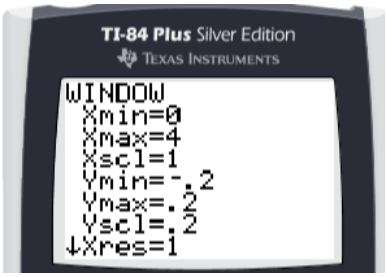
2. Consider f(x) where: $\boldsymbol{f(x) = x^4 - 8.65x^3 + 27.34x^2 - 37.2285x + 18.27}$

   a. What are the degree, leading coefficient and end behavior of this function?

   Degree:_____ Leading Coefficient: _____ End Behavior:_____

   b. Make a table of values for $-4 \leq x \leq 4$. How many zeros does the function appear to have from the table?

   c. Now change your window to this window here→

   d. What conclusions can you make from this new view of the graph?

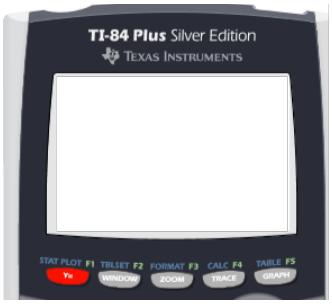
x	f(x)
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

3. The average annual price of gasoline can be modeled by the cubic function :

$$c(t) = 0.0007t^3 - 0.014t^2 + 0.08t + 0.96$$

where $c(t)$ is the *price* in dollars and t is the number of years since 1987.

a. Graph the function in your calculator using a domain of $0 \le t \le 30$. Sketch a picture of your graph:



b. Describe any extrema and end behavior.

c. This model was created in 2007. Using the model, predict the price of gasoline in 2014. How accurate is the model?

d. Going beyond the given domain in a model is called ***extrapolation***. Explain why extrapolation can be dangerous when predicting future events.

SAT Review!

MUTIPLE CHOICE

The number of honey bees in a particular hive can be modeled by the function ***b*** below:

$$b(t) = \frac{1}{2}t^2 - 20t + k$$

In the function, k is constant and $b(t)$ represents the number of bees on number t for $0 \le t \le 99$. On what number day would the number of honey bees in the hive be the same as it was on day number 10?

(A) 20
(B) 30
(C) 40
(D) 50
(E) 60

(Hint: Use the axis of symmetry!)

Free Response

Let the function g be defined by:

$$g(x) = x^2 + 18$$

If n is a positive number such that $g(2n) = 2 \cdot g(n)$, what is the value of n?