

6.3 Completing the Square

PRACTICE

Fill in the question mark so that the expression makes a perfect square trinomial.
Then write the expression as the perfect square.

$$1. x^2 + 24x + ?$$

$$144$$

$$(x + 12)^2$$

$$2. x^2 - 20x + ?$$

$$100$$

$$(x - 10)^2$$

$$3. x^2 + 30x + ?$$

$$225$$

$$(x + 15)^2$$

$$4. x^2 + 7x + ?$$

$$1\frac{49}{4}$$

$$(x + \frac{7}{2})^2$$

$$5. x^2 - 13x + ?$$

$$16\frac{9}{4}$$

$$(x - \frac{13}{2})^2$$

$$6. x^2 + x + ?$$

$$\frac{1}{4}$$

$$(x + \frac{1}{2})^2$$

Solve the equation by completing the square.

$$7. x^2 + 4x = 10$$

$$(x^2 + 4x + 4) = 10 + 4$$

$$(x + 2)^2 = 14$$

$$\sqrt{(x + 2)^2} = \sqrt{14}$$

$$x + 2 = \pm \sqrt{14}$$

$$\begin{array}{r} -2 & -2 \\ \hline x & = -2 \pm \sqrt{14} \end{array}$$

$$8. x^2 - 12x + 48 = 0$$

$$(x^2 - 12x + 36) = -48 + 36$$

$$(x - 6)^2 = -12$$

$$\sqrt{(x - 6)^2} = \sqrt{-12}$$

$$x - 6 = \pm i\sqrt{12}$$

$$\begin{array}{r} +6 & +6 \\ \hline x & = 6 \pm i\sqrt{12} \\ & = 6 \pm i\sqrt{4 \cdot 3} \\ & = 6 \pm 2i\sqrt{3} \end{array}$$

$$9. x^2 + 8x - 14 = 0$$

$$(x^2 + 8x + 16) = 14 + 16$$

$$(x + 4)^2 = 30$$

$$\sqrt{(x + 4)^2} = \sqrt{30}$$

$$x + 4 = \pm \sqrt{30}$$

$$\begin{array}{r} -4 & -4 \\ \hline x & = -4 \pm \sqrt{30} \end{array}$$

$$10. x^2 + 16x = 20$$

$$(x^2 + 16x + 64) = 20 + 64$$

$$(x + 8)^2 = 84$$

$$\sqrt{(x + 8)^2} = \sqrt{84}$$

$$x + 8 = \pm \sqrt{84}$$

$$\begin{array}{r} -8 & -8 \\ \hline x & = -8 \pm \sqrt{84} \\ & = -8 \pm 2\sqrt{21} \end{array}$$

$$11. 3x^2 + 36x + 162 = 0$$

$$3x^2 + 36x = -162$$

$$3(x^2 + 12x + 36) = -162 + 108$$

$$3(x + 6)^2 = -54$$

$$\sqrt{3(x + 6)^2} = \sqrt{-54}$$

$$x + 6 = \pm i\sqrt{18}$$

$$\begin{array}{r} -6 & -6 \\ \hline x & = -6 \pm 3i\sqrt{2} \end{array}$$

$$12. x^2 + 5x + 9 = 0$$

$$(x^2 + 5x + \frac{25}{4}) = -9 + \frac{25}{4}$$

$$(x + \frac{5}{2})^2 = -\frac{11}{4}$$

$$\sqrt{(x + \frac{5}{2})^2} = \sqrt{-\frac{11}{4}}$$

$$x + \frac{5}{2} = \pm i\sqrt{\frac{11}{4}}$$

$$\begin{array}{r} -\frac{5}{2} & -\frac{5}{2} \\ \hline x & = -\frac{5}{2} \pm \frac{\sqrt{11}}{2}i \end{array}$$

Write the quadratic function in vertex form.

13. $y = x^2 - 18x + 7$

$$y - 7 = x^2 - 18x + 81$$

$$y + 74 = (x - 9)^2$$

$$y = (x - 9)^2 - 74$$

14. $y = 2x^2 + 24x + 9$

$$y - 9 = 2x^2 + 24x$$

$$y - 9 = 2(x^2 + 12x + 36)$$

$$y + 63 = 2(x + 6)^2$$

$$y = 2(x + 6)^2 - 63$$

15. $f(x) = -x^2 - 9x + 8$

$$y - 8 = -x^2 - 9x$$

$$y - 8 = -(x^2 + 9x + \frac{81}{4})$$

$$y - \frac{113}{4} = -(x + \frac{9}{2})^2$$

$$y = -(x + \frac{9}{2})^2 + \frac{113}{4}$$

or

$$y = -(x + 4.5)^2 + 28.25$$

Write the quadratic function in vertex form. Find the roots. Graph it! Label the vertex and roots.

16. $y = x^2 - 6x + 4$

$$y - 4 = (x^2 - 6x + 9)$$

$$y + 5 = (x - 3)^2$$

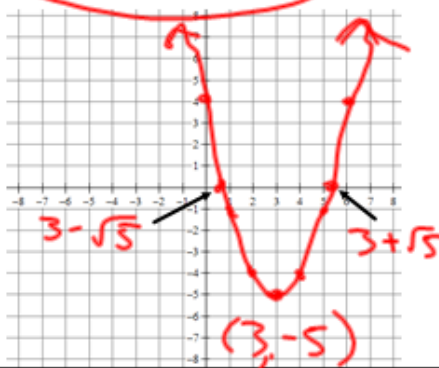
$$y = (x - 3)^2 - 5$$

$$0 = (x - 3)^2 - 5$$

$$\sqrt{5} = \sqrt{(x - 3)^2}$$

$$\pm\sqrt{5} = x - 3$$

$$3 \pm \sqrt{5} = x$$



17. $y = 2x^2 + 12x + 10$

$$y - 10 = 2x^2 + 12x$$

$$y - 10 = 2(x^2 + 6x + 9)$$

$$y + 8 = 2(x + 3)^2$$

$$y = 2(x + 3)^2 - 8$$

$$0 = 2(x + 3)^2 - 8$$

$$\frac{8}{2} = \frac{2(x + 3)^2}{2}$$

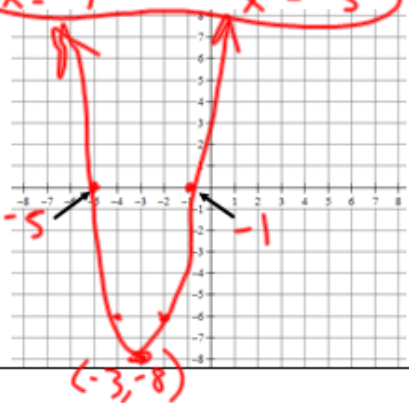
$$\sqrt{4} = \sqrt{(x + 3)^2}$$

$$\pm 2 = x + 3$$

$$-3 \pm 2 = x$$

$$x = -3 + 2 \quad x = -3 - 2$$

$$x = -1 \quad x = -5$$



18. $y = -x^2 + 10x - 22$

$$y + 22 = -x^2 + 10x$$

$$y + 22 = -(x^2 - 10x + 25)$$

$$y - 3 = -(x - 5)^2$$

$$y = -(x - 5)^2 + 3$$

$$0 = -(x - 5)^2 + 3$$

$$-3 = -(x - 5)^2$$

$$\sqrt{3} = \sqrt{(x - 5)^2}$$

$$\pm\sqrt{3} = x - 5$$

$$5 \pm \sqrt{3} = x$$

