1) $(x^3 + 14x^2 + 55x + 48) \div (x + 6)$

$$\begin{array}{r} x^2 + 8x + 7 + \frac{6}{x+6} \\ x+6 \overline{)\, x^3 + 14x^2 + 55x + 48} \\ x^3 + 6x^2 \\ \hline 8x^2 + 55x \\ -\ 8x^2 + 48x \\ \hline 7x + 48 \\ 7x + 42 \\ \hline 6 \end{array}$$

2) $(x^4 - 61x^2 - 26x + 16) \div (x - 8)$

$0x^3$!!

$$\begin{array}{r} x^3 + 8x^2 + 3x - 2 \\ x-8 \overline{)\, x^4 + 0x^3 - 61x^2 - 26x + 16} \\ -\ x^4 - 8x^3 \\ \hline 8x^3 - 61x^2 \\ -\ 8x^3 - 64x^2 \\ \hline 3x^2 - 26x \\ 3x^2 - 24x \\ \hline -2x + 16 \\ -2x + 16 \\ \hline 0 \end{array}$$

3) $(10a^4 + 60a^3 + 10a + 60) \div (a + 6)$

$0a^2$

$$\begin{array}{r} 10a^3 + 0a^2 + 0a + 10 \\ a+6 \overline{)\, 10a^4 + 60a^3 + 0a^2 + 10a + 60} \\ 10a^4 + 60a^3 \\ \hline 0a^3 + 0a^2 \\ 0a^3 + 0a^2 \\ \hline 0a^2 + 10a \\ 0a^2 + 0a \\ \hline 10a + 60 \\ 10a + 60 \\ \hline 0 \end{array}$$

$\boxed{10a^3 + 10}$

4) $(n^3 + 10n^2 + 30n + 72) \div (n + 7)$

$$\begin{array}{r} n^2 + 3n + 9 + \frac{9}{n+7} \\ n+7 \overline{)\, n^3 + 10n^2 + 30n + 72} \\ n^3 + 7n^2 \\ \hline 3n^2 + 30n \\ 3n^2 + 21n \\ \hline 9n + 72 \\ 9n + 63 \\ \hline 9 \end{array}$$

$n^2 + 3n + 9 + \frac{9}{n+7}$

Divide using synthetic division.

5) $(n^4 + 3n^3 - 9n - 38) \div (n + 3)$

$$\begin{array}{r|rrrrr} -3 & 1 & 3 & 0 & -9 & -38 \\ & & -3 & 0 & 0 & 27 \\ \hline & 1 & 0 & 0 & -9 & -11 \end{array}$$

$x^3 - 9 + \frac{-11}{n+3}$

6) $(a^4 - 4a^3 + 5a^2 + 8a - 14) \div (a - 2)$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 5 & 8 & -14 \\ & & 2 & -4 & 2 & 20 \\ \hline & 1 & -2 & 1 & 10 & 6 \end{array}$$

$x^3 - 2x^2 + x + 10 + \frac{6}{a-2}$

7) $(4a^3 - 36a^2 + 60a + 72) \div (a - 6)$

$$\begin{array}{r|rrrr} 6 & 4 & -36 & 60 & 72 \\ & & 24 & -72 & -72 \\ \hline & 4 & -12 & -12 & 0 \end{array}$$

$4x^2 - 12x - 12$

8) $(x^4 + 16x^3 + 75x^2 + 91x + 49) \div (x + 7)$

$$\begin{array}{r|rrrrr} -7 & 1 & 16 & 75 & 91 & 49 \\ & & -7 & -63 & -84 & -49 \\ \hline & 1 & 9 & 12 & 7 & 0 \end{array}$$

$x^3 + 9x^2 + 12x + 7$

Use the Factor Theorem to determine whether the given binomial is a factor of the given polynomial.

9) $(n^3 + 16n^2 + 71n + 56) \div (n + 8)$

$$\begin{array}{r|rrrr} -8 & 1 & 16 & 71 & 56 \\ & & -8 & 64 & -56 \\ \hline & 1 & 8 & 7 & 0 \end{array}$$

yes!

10) $(n^4 + 7n^3 - 25n^2 + 22n - 25) \div (n - 2)$

$$\begin{array}{r|rrrrr} 2 & 1 & 7 & -25 & 22 & -25 \\ & & 2 & 18 & -14 & 16 \\ \hline & 1 & 9 & -7 & 8 & -9 \end{array}$$

No!

11) $(m^4 - 17m^3 + 78m^2 - 47m - 63) \div (m - 7)$

$$\begin{array}{r|rrrrr} 7 & 1 & -17 & 78 & -47 & -63 \\ & & 7 & -70 & 56 & 63 \\ \hline & 1 & -10 & 8 & 9 & 0 \end{array}$$

yes!

12) $(v^4 - 6v^3 - 35v^2 + 26v + 20) \div (v + 4)$

$$\begin{array}{r|rrrrr} -4 & 1 & -6 & -35 & 26 & 20 \\ & & -4 & 40 & -20 & -24 \\ \hline & 1 & -10 & 5 & 6 & -4 \end{array}$$

NO!

Given a polynomial f(x) and a factor of f(x), factor f(x) completely.

13) $f(x) = 25x^3 - 40x^2 + 17x - 2;\ 5x - 2$

$$\begin{array}{r} 5x^2 - 6x + 1 \\ 5x-2 \,\big)\, 25x^3 - 40x^2 + 17x - 2 \\ -\ (25x^3 - 10x^2) \\ \hline -30x^2 + 17x \\ -30x^2 + 12x \\ \hline 5x - 2 \\ 5x - 2 \\ \hline 0 \end{array}$$

$(5x-2)(5x^2-6x+1)$

$\boxed{(5x-2)(5x-1)(x-1) = f(x)}$

14) $f(x) = 5x^3 - 18x^2 - 33x - 10;\ x - 5$

$$\begin{array}{r|rrrr} 5 & 5 & -18 & -33 & -10 \\ & & 25 & 35 & 10 \\ \hline & 5 & 7 & 2 & 0 \end{array}$$

$f(x) = (x-5)(5x^2+7x+2)$

$\boxed{f(x) = (x-5)(5x+2)(x+1)}$

15) $f(x) = 15x^3 - 28x^2 + 15x - 2;\ 3x - 2$

$$\begin{array}{r} 5x^2 - 6x + 1 \\ 3x-2 \,\big)\, 15x^3 - 28x^2 + 15x - 2 \\ 15x^3 - 10x^2 \\ \hline -18x^2 + 15x \\ -18x^2 + 12x \\ \hline 3x - 2 \\ 3x - 2 \\ \hline 0 \end{array}$$

$f(x) = (3x-2)(5x^2-6x+1)$

$\boxed{f(x) = (3x-2)(5x-1)(x-1)}$

16) $f(x) = 9x^3 + 3x^2 - 5x + 1;\ 3x - 1$

$$\begin{array}{r} 3x^2 + 2x - 1 \\ 3x-1 \,\big)\, 9x^3 + 3x^2 - 5x + 1 \\ 9x^3 - 3x^2 \\ \hline 6x^2 - 5x \\ 6x^2 - 2x \\ \hline -3x + 1 \\ -3x + 1 \\ \hline 0 \end{array}$$

$f(x) = (3x-1)(3x^2+2x-1)$

$f(x) = (3x-1)(3x-1)(x+1)$

$\boxed{f(x) = (3x-1)^2(x+1)}$