

A polynomial function is a function of the form:

$$f(x) = ax^n + bx^{n-1} + cx^{n-2} + \dots + dx + e$$

where:

a is called the _____ n is called the _____

When the function is written with terms in descending order of exponents, we say the function is in _____.

Examples:

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

	Example	Polynomial Function?	Standard form?	Leading Coefficient	Degree	Type?
1.	$f(x) = x^5 + \frac{2}{5}x^3 - 4$					
2.	$g(x) = x^2 - 42x^3 + 5$					
3.	$h(n) = 0.23n^4 - 5n^2$					
4.	$k(v) = 4 - 3v$					
5.	$s(x) = 18x^{-2} - 12x^5 - 21x^3$					
6.	$t(p) = 3^p - 21p^3$					
7.	$v(x) = 7x^0$					
8.	$z(r) = 7r - 2^{-2} - \pi r^2$					

Use direct substitution to evaluate $g(x) = 2x^3 - 4x^2 + 5x$ when $x = -2$

Use direct substitution to evaluate $t(a) = -a^5 - a^2 + a$ when $a = -1$

Synthetic Substitution

Synthetic substitution is a method for evaluating a polynomial that uses fewer steps.

- Step 1: Write the value to be evaluated outside and the coefficients in descending order inside.
- Step 2: Bring down the leading coefficient and multiply by the number on the left.
- Step 3: Write the product from the last step under the second coefficient. Add and bring down.
- Step 4: Multiply the sum from the last step by the number on the left.
- Step 5: Repeat for remaining coefficients. The final sum is the value of $f(x)$.

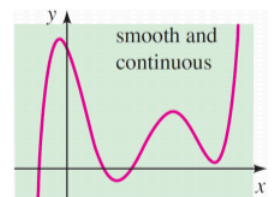
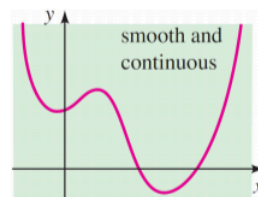
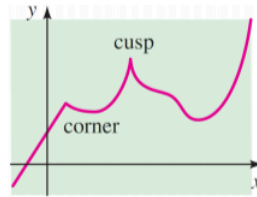
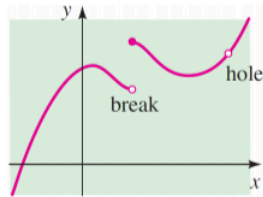
Examples:

Use synthetic substitution to evaluate $g(p) = 2p^3 - 4p^2 + 5p$ when $p = -2$

Use synthetic substitution to evaluate $t(x) = -x^5 - 7x^3 + 3x^2 - 2$ when $x = -1$

Graphing Polynomials using a Table of Values

The graph of a polynomial function is always a smooth curve; that is, it has no breaks or corners:

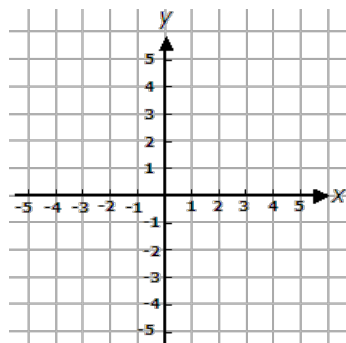


These graphs are **NOT** _____!

These graphs are **ARE** _____!

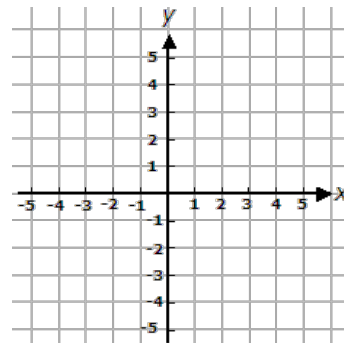
Examples:

Graph $f(x) = -x^3 + x^2 + 3x - 3$



x	f(x)
-3	
-2	
-1	
0	
1	
2	
3	

Graph $g(n) = n^4 - n^3 - 4n^2 + 4$



n	g(n)
-3	
-2	
-1	
0	
1	
2	
3	

Practice 7.2: Polynomials

Write each polynomial in standard form, if not already. Then tell the degree, leading coefficient and name the type of polynomial.

1) $2x^5$

2) $-2 + 4x^2 + 2x$

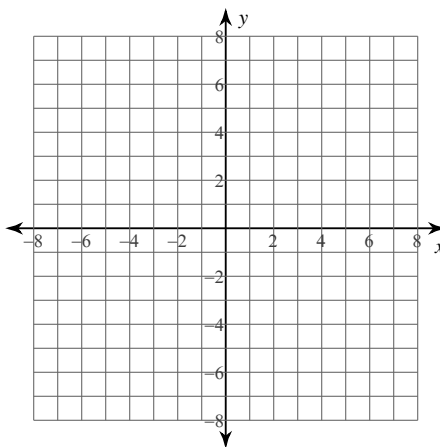
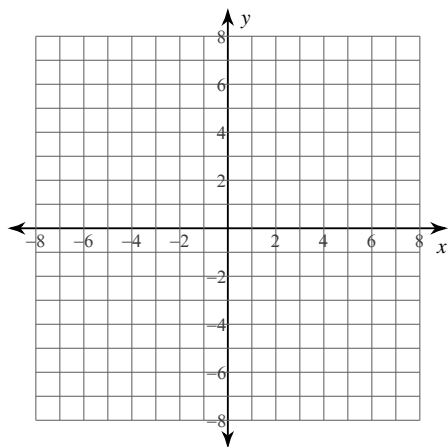
3) $-4b^3 + 5b$

4) $-5 + 10a$

Sketch the graph of each function by making a table of values. Although it is not necessary, you may use your calculator to help guide you.

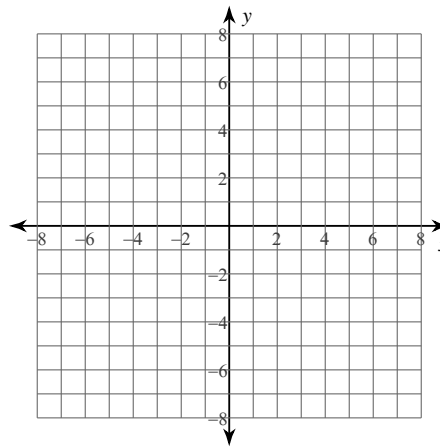
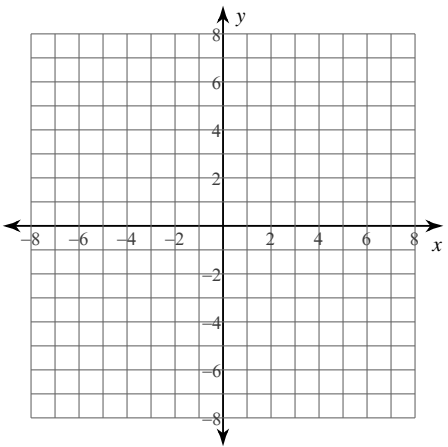
5) $f(x) = x^3 - 3x^2$

6) $f(x) = x^4 - x^3 - 4x^2 + 6$



7) $f(x) = -x^4 + x^3 + 4x^2 - 5$

8) $f(x) = -x^3 + 4x^2 - 7$



Evaluate each function at the given value using direct substitution.

9) $f(n) = -4n^3 - 21n^2 + 32$ at $n = -5$

10) $f(a) = a^4 - 2a^3 - 15a^2 - 3a + 8$ at $a = 5$

$$11) f(a) = -6a^3 + 32a^2 - 12a - 2 \text{ at } a = 5$$

$$12) f(a) = a^4 + a^3 - 3a^2 + 3a + 13 \text{ at } a = -2$$

Evaluate each function at the given value using synthetic substitution.

$$13) f(m) = m^3 - 10m^2 + 25m + 2 \text{ at } m = 6$$

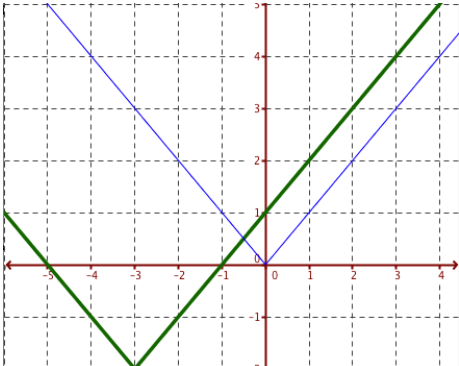
$$14) f(x) = x^3 + 12x^2 + 34x - 21 \text{ at } x = -6$$

$$15) f(n) = -3n^3 - 4n^2 + 2n + 10 \text{ at } n = -1$$

$$16) f(n) = n^4 + 9n^3 + 20n^2 - 6n - 28 \text{ at } n = -5$$

$$17) f(a) = -a^6 - a^5 + 34a^4 - 20a^3 + 6a - 40 \text{ at } a = 5$$

$$18) f(m) = m^5 - 12m^3 + 16m^2 + 5m + 6 \text{ at } m = -4$$

<i>Algebra Skillz</i>		
GRAPH	SIMPLIFY	SOLVE
<p>Below, the graph of $f(x) = x + 3 - 2$ is sketched in bold. Its parent function $f(x) = x$ is represented by the thin curve.</p> <p>1. Describe the translation of the parent graph.</p> <p>2. How does the translation relate to the equation?</p> 	<p>3. $\sqrt{18} + \sqrt{45} + \sqrt{54}$</p> <p>4. $-2(12 + 2\sqrt{20})$</p>	<p>5. Solve: $(2x - 1)(3x + 2) = 0$</p> <p>6. Factor and solve. $x^2 - 36x + 35 = 0$</p>

Application 7.2

- Use synthetic substitution to evaluate $r(p) = 2p^3 - 4p^2 + 5p$ when $p = -1$.
- Name the leading coefficient and degree of $t(x) = 23 - 7x^3 - x^5 + 3x^2$
- Sully loves baseball. One night after BINGO, Sully looked at the average salary S (in thousands of dollars) for major league baseball players from 1994 to 2003. He figures out that he can model the data fairly accurately using the following polynomial:

$$S(x) = -4.10x^3 + 67.4x^2 - 121x + 1170$$

where x is the number of years since 1994.

Number of Years since 1994 (x)	Average Salary $S(x)$
0	
1	
2	
3	
5	
7	
9	

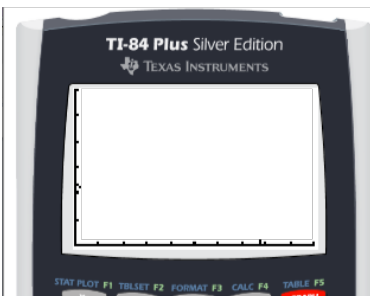
- Fill in the table to the right.
- Explain what $x = 2$ means in the context of this problem.
- What value of x would represent the year 2000?
- Sully puts the polynomial into his calculator in $Y=$. Then, he views the function using a standard window. (Hint: press $ZOOM$ 6 .) Describe how the function looks on his calculator and record the window to the right:

WINDOW	
Xmin=	<input type="text"/>
Xmax=	<input type="text"/>
Xscl=	<input type="text"/>
Ymin=	<input type="text"/>
Ymax=	<input type="text"/>
Yscl=	<input type="text"/>

X	Y1
0	<input type="text"/>
1	<input type="text"/>
2	<input type="text"/>
3	<input type="text"/>
5	<input type="text"/>
7	<input type="text"/>
9	<input type="text"/>

- That obviously didn't work well for Sully. He calls his Algebrother Bean and asks "What's the dillio?". Bean tells him his function is off the screen and he needs to fix his window. "How do I know what numbers to put in for the window?" Sully asks. Bean replies, "Use the **table** to estimate, brah!" Sully goes to the table to get a clue. Fill in the table to the left based on what Sully sees. (Hint: press $2nd$ $GRAPH$)
- Confirm your answers to part (a) above.

g. It's now time for Sully to clean up his window. He uses the values from the table to construct an appropriate window. He figures a good domain for this problem would be 0 to 20 years so he fixes his window such that $Xmin=0$, $Xmax=20$, with $Xscl=2$. Using a $Yscl=500$, create a window that would display a range that would accommodate the values in the table above. (Hint: use $0 \leq y \leq 3000$) Sketch your function:

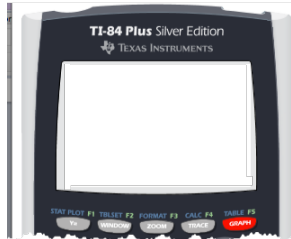
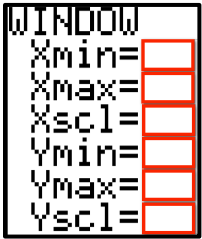


- Estimate when baseball players will be playing for free using the trace button.
- Can we trust this estimate? Why or Why not?

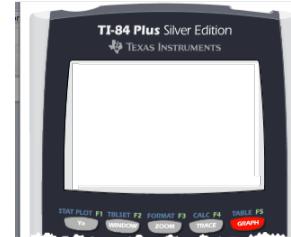
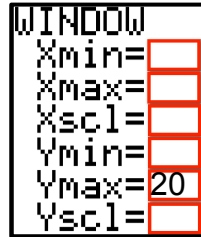
7.2 – Exponent Properties

4. Congrats! You have learned how to set the window in your calculator! Graph each polynomial below and give an appropriate window. Then, sketch the graph in the window.

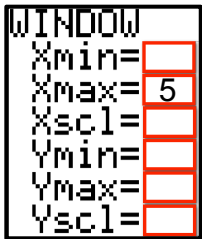
a. $f(x) = x^3 + 4x^2 - 8x + 11$



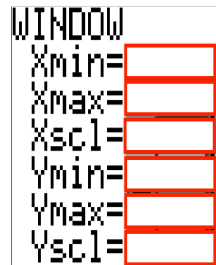
b. $f(x) = x^5 - 7x^4 + 25x^3 - 40x^2 + 13x$



c. $f(x) = -x^4 + 3x^3 + 15x$



d. $f(x) = -x^3 + 36x^2 - 10$



$-10000 \leq x \leq 10000$

MUTIPLE CHOICE

The number of honey bees in a particular hive can be modeled by the function b below:

$$b(t) = \frac{1}{2}t^2 - 20t + k$$

In the function, k is constant and $b(t)$ represents the number of bees on number t for $0 \leq t \leq 99$. On what number day would the number of honey bees in the hive be the same as it was on day number 10?

- (A) 20
- (B) 30
- (C) 40
- (D) 50
- (E) 60

Free Response

Let the function g be defined by:

$$g(x) = x^2 + 18$$

If n is a positive number such that $g(2n) = 2g(n)$, what is the value of n ?

