9. 4 – Intro to Logarithms

Name:_______________________

Two areas of application for logarithms are how we measure earthquakes and sound. What are those measurements called?
1. 
2. 

**Definition of a Logarithm**

\[ y = \log_b x \] if and only if \[ b > 0, b \neq 1, x > 0 \]

How do you say \( \log_b x \)? “_________________________”

**Practice: Rewrite in exponential form**

1. \( \log_3 243 = 5 \) 
2. \( \log_5 125 = 3 \) 
3. \( \log_9 1 = 0 \) 
4. \( \log_{\frac{1}{2}} 16 = -4 \)

**Practice: Rewrite in logarithmic form**

5. \( 3^2 = 9 \) 
6. \( 2^5 = 32 \) 
7. \( 5^0 = 1 \) 
8. \( \frac{3}{\sqrt{64}} = 4 \)

**Evaluate the following logarithms (without a calculator):**

9. \( \log_2 8 \) 
10. \( \log_7 \frac{1}{7} \) 
11. \( \log_{13} 81 \) 
12. \( \log_{25} 5 \)

**Common Logarithm:** \( \log_{10} x \) \( \text{write as} \) \( \text{________} \)

**Natural Logarithm:** \( \log_e x \) \( \text{write as} \) \( \text{________} \)

**Evaluate using a calculator:**

13. \( \log 13 \approx \) 
14. \( \ln 6 \approx \)

**Things that simplify...**

1. \( \log_b b^x = x \)
2. \( b^{\log_b x} = x \)

**Simplify:**

15. \( 10^{\log 6} \) 
16. \( \log_3 9^x \) 
17. \( e^{\ln 3x} \) 
18. \( \ln e^x \)
9.4 – Intro to Logarithms

**GRAPHING LOGARITHMIC FUNCTIONS:**

\[ g(x) = b^x \text{ where } b > 1 \]

Graph \( f(x) = \log_b x \) above by reflecting \( g(x) \) across the line \( y = x \).

\[ g(x) = b^x \text{ where } 0 < b < 1 \]

Graph \( f(x) = \log_b x \) above by reflecting \( g(x) \) across the line \( y = x \).

19. Graph \( y = \log_4 x \)

\[ x \quad y \]
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9.4 Practice - Intro to Logarithms

Rewrite each equation in exponential form.

1) \( \log_{14} 196 = 2 \)

2) \( \log_{18} 324 = 2 \)

3) \( \log_{243} 3 = \frac{1}{5} \)

4) \( \log_{3} 243 = 5 \)

5) \( \log_{7} 49 = 2 \)

6) \( \log_{11} \frac{1}{121} = -2 \)

Rewrite each equation in logarithmic form.

7) \( 225^{-\frac{1}{2}} = \frac{1}{15} \)

8) \( 19^2 = 361 \)

9) \( 7^2 = 49 \)

10) \( 3^2 = 9 \)

11) \( 18^{-2} = \frac{1}{324} \)

12) \( 9^{-\frac{1}{2}} = \frac{1}{3} \)

Evaluate each expression.

13) \( \log_{4} 16 \)

14) \( \log_{6} \frac{1}{216} \)
15) \( \log_2 16 \)

16) \( \log_2 32 \)

17) \( \log_{16} \frac{1}{2} \)

18) \( \log_6 \frac{1}{36} \)

Sketch the graph and identify the domain and range of each.

19) \( f(x) = \log_4 (x - 1) - 5 \)

20) \( f(x) = \log_3 (x + 5) - 5 \)

21) \( f(x) = \log (x - 1) + 5 \)

22) \( f(x) = \log_2 (x + 1) + 3 \)
9.4 Application and Extension

1. a. Evaluate \( \log_3 27 \)  
   b. Evaluate \( \log_{\frac{1}{4}} 1 \)

2. Most tornadoes last less than an hour and travel less than 20 miles. The wind speed \( w \) (in miles per hour) near the center of a tornado is related to the distance \( d \) (in miles) the tornado travels. The following model shows this relationship:

\[
w = 93 \log d + 65
\]

a. If a tornado has traveled 13.6 miles, what is its wind speed?

b. If a tornado’s wind speed is 207.3 mph, how far did it travel? \( \text{(hint: use a graphing calculator and graph both sides of the equation.)} \)

For 3-6, the graph of \( f(x) = \log_2 x \) is given on the right along with three coordinate points. For each problem, translate \( g(x) \) by using \( f(x) \) as the “parent” function.

3. \( g(x) = 2 \log_2 x \)

4. \( g(x) = -\log_2 x \)

5. \( g(x) = \log_2 (-x) \)

6. \( g(x) = \frac{1}{2} \log_2 x - 3 \)
7. When Mr. Sullivan has indigestion and flatulence, the disturbance can often feel like an earthquake. On different days, Sully’s students measured the “disturbance” and recorded the information (see table).

<table>
<thead>
<tr>
<th>Minutes after lunch</th>
<th>Magnitude measured on the Ripped It Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3.1</td>
</tr>
<tr>
<td>10</td>
<td>3.6</td>
</tr>
<tr>
<td>20</td>
<td>4.2</td>
</tr>
<tr>
<td>35</td>
<td>4.4</td>
</tr>
</tbody>
</table>

a. Using your calculator, find a logarithmic regression model that matches the data. Round to three decimal places. *(Hint: Enter the data into two lists in the calculator, then hit “STAT” and then “CALC” menu. One of the options will be “LnReg.”)*

b. Using your model, estimate the disturbance of an event 45 minutes after Sully’s lunch.

c. In Sully’s class after lunch, a student is surprised to feel a disturbance of 4.1 magnitude. How long has it been since lunch ended? *(Round your answer to three decimals.)*

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### Algebra Skills:

1. Below are graphs of \( f(x) = |x| \) (thin line) and its translation (bold line). Write an equation of the translation.

2. Simplify the fraction by rationalizing the denominator.

   2. \( \frac{5}{\sqrt{10}} \)

3. \( \frac{3}{2\sqrt{6}} \)

4. Solve by factoring.

   4. \( 5x^3 - 10x^2 - 175x = 0 \)

5. \( 18x^2 - 15x + 3 = 0 \)

### SAT Prep:

1. Simplify: \( (2^3x)(2^{5-x}) \)

   (A) \( 2^{15x-3x^2} \)

   (B) \( 2^{5-3x^2} \)

   (C) \( 2^{2x+5} \)

   (D) \( 2^{5-2x} \)

2. If \( f(x) = 2(4)^{x+4} \), find \( f(-5) \).

   2. \( f(-5) = 2(4)^{-5+4} = 2(4)^{-1} = \frac{2}{4} = 0.5 \)