7.4 – Factor and Solving Polynomials

Recall Factoring:

Factoring out a GCF:  Factoring trinomials:

a. \( \_ \)  b. \( \_ \)

Factoring out a GCF, then trinomial:  Factoring Special Cases:

c. \( \_ \)  d. \( \_ \)

Factoring by Grouping

Sometimes if you have a polynomial with no common factor in EVERY term, factor by grouping can work....

Examples:

a. \( \_ \)  b. \( \_ \)

c. \( 10r^3 + 6r^2 - 5r - 3 \)  d. \( 28x^3 + 49x^2 - 16x - 28 \)

Factoring Polynomials in Quadratic Form

Examples:

a. \( \_ \)  b. \( \_ \)

c. \( x^3 + 7x^2 - 9x - 63 \)  d. \( 16g^4 - 625 \)
Factoring with Cube Patterns

Sum of Two Cubes
\[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]
64x^3 + 27 =

Difference of Two Cubes
\[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]
8x^3 - 125 =

Examples:
\[ a. \ 3y^5 - 75y^3 \]
\[ b. \ 16b^5 + 686b^2 \]

CHOOSE THE APPROPRIATE METHOD!!!!

a. 

b. 

c. \[ z^5 - 3z^4 - 16z + 48 \]

d. \[ 32w^5 - 108w^2 \]

Solving Polynomial Equations
We can use the zero product property to solve polynomial equations as well:

a. 

b. 

c. \[ y^3 - 5y^2 = 0 \]

d. \[ d^6 - 4d^4 - 9d^2 + 36 = 0 \]
Factor completely by factoring out a GCF, then factoring the remaining trinomial.

1) \( x^3 + x^2 - 6x \)  
2) \( 2x^4 - 12x^3 + 18x^2 \)

3) \( 10x^4 - 90x^2 \)  
4) \( x^3 - 7x^2 + 12x \)

Factor each sum of cubes.

5) \( 27x^3 + 125 \)  
6) \( 8x^3 + 27 \)

Factor each difference of cubes.

7) \( 8x^3 - 1 \)  
8) \( 27x^3 - 125 \)

Factor each completely by grouping.

9) \( x^3 + 5x^2 - 6x - 30 \)  
10) \( 7r^3 - 42r^2 - 3r + 18 \)

11) \( 5n^3 + 40n^2 - n - 8 \)  
12) \( 6x^3 - x^2 - 42x + 7 \)

Factor each quadratic form polynomial completely.

13) \( x^4 + 6x^2 - 16 \)  
14) \( m^4 - 1 \)
15) $5a^5 + 55a^3 + 150a$

16) $4x^5 - 16x^3 + 12x$

Hint: Take out a GCF!!

Solve for $x$.

17) $x^3 - 2x^2 - 5x + 10 = 0$

18) $x^4 - 7x^2 - 18 = 0$

19) $(3x - 5)(x - 4) = 0$

20) $9x^4 - 30x^2 + 25 = 0$

21) $8x^4 - 54x^2 + 81 = 0$

22) $x^3 - 2x^2 + x = 0$

This problem is optional. Only the Jedi Knights of factoring should attempt it.

23) $x^9 - 25x^5 + 144x = 0$
7.4 – Factor and Solving Polynomials

Application 7.4

1. Factor: \( z^5 - 3z^4 - 16z + 48 \) 

2. Solve: \( 48y^5 = 27y^3 \)

Find the possible value(s) of \( x \).

3. a. Area = 48 
   \((3x + 2) (x+4)\) 
   Rectangle

   b. Volume = 40 
   \(2x \times x-1 \times x-4\) 

   c. Volume = 125\(\pi\) 
   \(\frac{1}{3} \pi r^2 h\) 

4. Ramstein HS decides that the foyer needs a giant bust of Mr. Brust's head: a "Bust-o-Brust," you could say. The Bust-o-Brust is to be made from 250 cubic inches of clay in the shape of a rectangular prism (see # 3b above). The height and the width of the prism each have to be 5 inches less than the length. Draw a picture and solve a polynomial equation to find the dimensions of the prism.
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**Algebra Skillz**

**GRAPH**
Below, the graph of \( f(x) = |x - 1| + 2 \) is sketched in bold. Its parent function \( f(x) = |x| \) is represented by the thin curve.

1. Describe the translation of the parent graph.

2. How does the translation relate to the equation?

**SIMPLIFY**

3. \( \sqrt{25} + \sqrt{40} + \sqrt{90} \)

4. \( \sqrt{6(12 - 2\sqrt{2})} \)

**SOLVE**

5. Solve:
\( x^2(x + 14) = 0 \)

6. Factor and solve.
\( x^2 - 25x + 24 = 0 \)

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**SAT Review**

**MULTIPLE CHOICE**

For what value of \( x \) is the statement below false?

\[ 5x^2 < (5x)^2 \]

(A) -5  
(B) 0  
(C) \( \frac{1}{5} \)  
(D) 1  
(E) For no value of \( x \)

**Free Response**

Let \([x] \) be defined as \([x] = x^2 - x \) for all values of \( x \).

If \([a] = [a - 2] \), what's the value of \( a \)?